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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

W6519

A NONLINEAR PROGRAMING MODEL FOR  
OPTIMIZED SORTIE ALLOCATION

by

Klaus Paul Wirths

\*\*\*

March 1989

Thesis Advisor: Alan R. Washburn

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The United States Air Force uses a nonlinear programming model to assess the utilization of weapons and sorties needed to achieve a maximum value of destroyed targets in a multi-period, Theater-Level conflict. The current model is modified by constraining the consumption of weapons. Alternate objective functions are introduced. Their meaning and influence on the optimization is compared. An increase in the worth of destroyed targets is gained if the model can more flexibly utilize weapons than is currently the case. The optimization can be further improved if all time periods are considered simultaneously while assigning sorties to targets, rather than the current myopic approach.

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A Nonlinear Programing Model for Optimized Sortie Allocation

by

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Submitted in partial fulfillment of the  
requirements for the degree of

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Kneale T. Marshall  
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## ABSTRACT

The United States Air Force uses a nonlinear programming model to assess the utilization of weapons and sorties needed to achieve a maximum value of destroyed targets in a multi-period, Theater-Level conflict. The current model is modified by constraining the consumption of weapons. Alternate objective functions are introduced. Their meaning and influence on the optimization is compared. An increase in the worth of destroyed targets is gained if the model can more flexibly utilize weapons than is currently the case. The optimization can be further improved if all time periods are considered simultaneously while assigning sorties to targets, rather than the current myopic approach.

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The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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## 1. INTRODUCTION

In 1988 the United States Air Force purchased over \$ 2 billion worth of weapons for use in different theaters around the world. The projected need for the quantity of different weapon types is based on an annual Nonnuclear-Weapon Consumables Analysis (NCAA) performed by the Directorate of Plans, USAF [Ref. 1]. Unlike other services, the USAF relies widely on mathematical programming models in order to optimize the allocation of weapons.

In 1974 RAND developed a nonlinear programming model that optimizes the number of different sortie types assigned to several target types by maximizing the military worth of killed targets [Ref. 2: p. 5]. Since each target type was given a different target value, the model attempts to assign sorties to maximum value targets first. To avoid an undesired concentration of sortie allocations to a few or even one target type, a nonlinear objective function was introduced. Within the model only the number of available targets and sorties are constrained. The expenditure of weapons is not considered. The number of targets one sortie is able to destroy is expressed by an effectiveness parameter that depends only on sortie and target type.

The required input data structure for the RAND-model is a simplification of the much more complex data base contained in the Joint Munitions Effectiveness Manual (JMEM) used by USAF. The JMEM data base determines effectiveness as a function of weather and mission profile (tactic) as well as type of aircraft and type of target. In the current operation a model called SELECTOR sorts the JMEM data base so that for each sortie-target type combination, all feasible tactics are ordered from the most to the least cost-effective, including the cost of aircraft attrition. This list is referred to as the Preferred Weapon List.

The data in the Preferred Weapon List must be reduced to input parameters depending only on sortie and target type as mentioned earlier. This is basically done by selecting the most cost-effective tactic from the list feasible for weather situations considered in the model. After the optimization has determined the optimal number of sorties assigned to different targets, the number of remaining targets and the expenditure of weapons is evaluated. This process is repeated in subsequent time periods with a new inventory of sorties and also by recording the remaining number of active targets and weapons available. In this way, tactical changes in a given scenario over time are

considered by optimizing sequentially for discrete time periods. This process is accomplished in one programming model and is called the HEAVY ATTACK model. The USAF interest is mainly in the consumption of weapons utilized over all time periods.

The objectives of this Thesis are to include a weapons constraint in the RAND-model and to investigate alternatives to the currently used objective function. In addition, the RAND-model is expanded so that more available information is included in the optimization in order to gain a higher total military worth of killed targets than is currently achieved. Therefore, the consumption of weapons used by less cost-effective tactics is investigated when other weapons, used by the most cost-effective tactic, are exhausted. As a final consideration, one global optimization over all time periods is compared to the current sequential optimization method. Global optimization achieves a higher overall worth of killed targets. However, gaining a higher military worth of killed targets serves only as an aid in analyzing the predicted need of weapons. The value of the revisions suggested in this Thesis have to be measured on their ability to satisfy the demands of the USAF and simultaneously meet budget constraints.

## II. BASIC STRUCTURE OF HEAVY ATTACK

### A. THE ORIGINAL RAND - MODEL

In 1974 RAND developed a nonlinear programming model whose objective was to determine the optimal number of sorties of type  $i$  assigned to targets of type  $j$  by maximizing the total military value of destroyed targets. The relationship between an assigned sortie and a target kill is established by introducing "sortie effectiveness"  $\bar{E}_{i,j}$ . The parameter  $\bar{E}_{i,j}$  defines the average number of kills that one sortie of type  $i$  will achieve when it is assigned to targets of type  $j$ .

#### Definition of index

$i$	sortie type
$j$	target type

#### Parameter

$T_j$	total number of type $j$ targets available at the beginning of a time period
$V_j$	military worth of type $j$ target
$S_i$	total number of type $i$ sorties available
$\bar{E}_{i,j}$	average number of type $j$ targets killed by one type $i$ sortie

#### Variables

$SX_{i,j}$	number of type $i$ sorties assigned to type $j$ targets
------------	---

## Model

$$\text{Max } z = \sum_j V_j \times f_j \left( \sum_i \bar{E}_{i,j} \times SX_{i,j} \right)$$

s.t.

$$\sum_j SX_{i,j} \leq S_i \quad \forall i$$

$$f_j \left( \sum_i \bar{E}_{i,j} \times SX_{i,j} \right) \leq T_j \quad \forall j$$

$$\sum_{j \in J} SX_{i,j} \leq c \times \sum_j SX_{i,j} \quad \forall i$$

where  $J$  is a subset of all targets of type  $j$  and  $0 < c < 1$ .

$$0 \leq SX_{i,j} \quad \forall i, j$$

$f_j(\sum_i \bar{E}_{i,j} \times SX_{i,j})$  is a concave function that approaches 1 for large arguments. The RAND - model (and HEAVY ATTACK) utilizes a specific analytic from that will be examined in detail later. The recipe constraints  $\sum_{j \in J} SX_{i,j} \leq c \times \sum_j SX_{i,j}$  limit the number of sorties of type  $i$  which are assigned to a list of targets by a fraction of the total number of sorties of type  $i$ . Since these constraints are not used by the USAF in their current weapon analysis, this inequality will be omitted from now on in the Thesis.

## B. THE ROLE OF SELECTOR

Based on the information contained in the JMEM the effectiveness of a sortie depends on sortie type, target type, weapon type , weather and tactics or mission profile.

### Definition of index

i	sortie type
j	target type
k	weapon type
w	weatherband index
r	index for used tactic

### Definition of parameter

$E_{i,j,r,w}$	number of type j targets killed by one type i sortie using tactic r in weatherband w
$B_{i,j,r,w}$	number of weapons carried by one type i sortie which is assigned to type j target in weatherband w and using tactic r
$K_{i,j,r,w}$	type of weapon which is loaded on sortie i and will be deployed to target j by using tactic r in weatherband w

The JMEM data have too many subscripts to match the required input data structure of the RAND - model. The number of subscripts of a sortie needs to be reduced so that  $E_{i,j}$  depends only on sortie and target type. The first part of the task of reducing the number of subscripts from 4 to 2 is accomplished by the sorting program SELECTOR. The output data of SELECTOR - referred to as Preferred Weapon List - contains for each different sortie - target type combination five distinct items:

1. The worst weatherband in which a tactic can be used.
2. The types of weapons that can be allocated.
3. The relative cost-efficiency of a tactic given by its order on the list.
4. The number of targets which can be killed by one sortie.
5. The number of weapons that can be carried by one sortie for each weapon type (mixes of weapons are not considered).

The data structure of the Preferred Weapon List, which will be used later for the aggregation of the input data  $\bar{E}_{i,j}$  for the RAND - model, is illustrated by the following example:

#### Subset of data from Preferred Weapon List

i	j	r	w	$K_{i,j,r,w}$	$E_{i,j,r,w}$	$B_{i,j,r,w}$
1	29	1	4	3	0.137	4
1	29	2	3	1	0.664	6
1	29	3	2	17	1.580	2
1	29	4	5	17	1.600	2

For example, the most cost-efficient and feasible tactic for weatherband  $w=3$  is tactic  $r=2$ . Tactic  $r=1$  is more cost-efficient because it is first on the list, but is only feasible in weatherband  $w=4$  or higher. Weatherband  $w=1$  expresses best weather while weatherband  $w=6$  represents the worst weather. Tactic  $r=3$  is feasible (a tactic feasible in  $w$  is always feasible in better weatherbands) but less cost-efficient than tactic  $r=2$ .

The given data can be represented in the following way:

Table 1.  $E_{i,j,r,w}$  - VALUES: Number of targets of type  $j$  killed by one sortie of type  $i$  using tactic  $r$  in weatherband  $w$ .

i	j	r	w=1	w=2	w=3	w=4	w=5	w=6
1	29	1	0	0	0	0.137	0.137	0.137
1	29	2	0	0	0.664	0.664	0.664	0.664
1	29	3	0	1.580	1.580	1.580	1.580	1.580
1	29	4	0	0	0	0	1.600	1.600



Table 2.  $B_{i,j,r,w}$  - VALUES: Number of weapons that are loaded on one sortie of type i which is assigned to target type j and using tactic r in weatherband w.

i	j	r	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
1	29	1	0	0	0	4	4	4
1	29	2	0	0	6	6	6	6
1	29	3	0	2	2	2	2	2
1	29	4	0	0	0	0	2	2

Table 3.  $K_{i,j,r,w}$  - VALUES: Type of weapon that is allocated to a sortie of type i which is assigned to a target of type j and using tactic r in weatherband w

i	j	r	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
1	29	1	0	0	0	3	3	3
1	29	2	0	0	1	1	1	1
1	29	3	0	17	17	17	17	17
1	29	4	0	0	0	0	17	17

Since HEAVY ATTACK only considers the tactic at the top of the list for each weatherband, and since weapon type is implied by tactics, SELECTOR essentially reduces the number of subscripts from 4 to 3.

### C. DETERMINATION OF $\bar{E}_{i,j}$ IN HEAVY ATTACK

An important assumption for HEAVY ATTACK in order to understand the logic behind the aggregation of  $\bar{E}_{i,j}$  is that the weather is *not known* at the time when sorties are assigned to targets. This leads to the condition that the effectiveness of a sortie and the consumption of weapons in a particular weatherband has to be proportional to the probability that this weather will occur.

This probability is represented in HEAVY ATTACK by a given distribution of 6 distinct weatherbands:

$$PR_w = \text{probability that weatherband } w \text{ will occur at a certain time in the future,}$$

$$w = 1, 2, \dots, 6.$$



Throughout this Thesis the following distribution is used:

**Table 4. WEATHER DISTRIBUTION IN HEAVY ATTACK:** Probability that weatherband  $w$  occurs when sorties are allocated to targets.

	$w = 1$	$w = 2$	$w = 3$	$w = 4$	$w = 5$	$w = 6$
$PR_w$	0	0.02	0.14	0.07	0.07	0.70

Since weatherband  $w = 1$  will never occur, the effectiveness for any sortie in this weatherband is irrelevant. It is assumed that any weapon which is feasible for a certain sortie - target combination can be used in the weatherband determined by SELECTOR or in any better weather (higher weatherband).

HEAVY ATTACK uses for each weatherband only the top weapon on Preferred Weapon List. This means that the model will allocate the most cost-efficient weapon feasible in each weatherband. Therefore the data set  $E_{i,j,r,w}$  can be reduced by the subscript  $r$  such that:

$E_{i,j,w}^*$  = the effectiveness of the most cost-efficient tactic in weatherband  $w$ .

**Table 5. EFFECTIVENESS OF THE MOST COST - EFFICIENT TACTIC:** In each weatherband  $w$  the first effectiveness value in Table 1 greater than zero is selected.

	$w = 1$	$w = 2$	$w = 3$	$w = 4$	$w = 5$	$w = 6$
$E_{i,j,w}^*$	0	1.580	0.664	0.137	0.137	0.137

Applying the same reasoning on the data set  $B_{i,j,r,w}$  and  $K_{i,j,r,w}$  yields :

$B_{i,j,w}^*$  = number of weapons used by the most cost-efficient tactic in weatherband  $w$ ,

$K_{i,j,w}^*$  = type of weapon used by the most cost-efficient tactic in weatherband  $w$ .

Table 6. **WEAPON LOAD OF THE MOST COST - EFFICIENT TACTIC:** In each weatherband the first weapon load value in Table 2 greater than zero is selected.

	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
$B_{i,j,w}^*$	0	2	6	4	4	4

Table 7. **WEAPON TYPE OF THE MOST COST - EFFICIENT TACTIC:** In each weatherband w the first weapon type in Table 3 not equal to zero is selected.

	w = 1	w = 2	w = 3	w = 4	w = 5	w = 6
$K_{i,j,w}^*$	0	17	1	3	3	3

Since each weatherband will occur with the probability  $PR_w$ , the averaged effectiveness must be

$$\bar{E}_{i,j} = \sum_w PR_w \times E_{i,j,w}^* = 0.240$$

In general the process of obtaining  $\bar{E}_{i,j}$  is a little more complicated than described above because HEAVY ATTACK is permitted to use tactics lower than first order when first order weapon types have been exhausted. This can happen because HEAVY ATTACK is actually a model of protracted war. First order tactics are preferred because they represent the most cost-effective tactic. The war may last for several periods (4 in this Thesis), and it is possible that certain tactics may not be feasible in later periods on

account of weapon exhaustion. Suppose for example, that weapon type 3 has been exhausted in a previous time period and is therefore no longer available. The top weapon for weatherband  $w=4, 5$  or  $6$  is now weapon type 1. The new effectiveness values  $E_{i,j,r,w}$  are:

**Table 8.  $E_{i,j,r,w}$  - VALUES AFTER WEAPON  $k=3$  IS EXHAUSTED:** Number of targets of type  $j$  killed by one sortie of type  $i$  using tactic  $r$  in weatherband  $w$  that is applicable.

$i$	$j$	$r$	$w=1$	$w=2$	$w=3$	$w=4$	$w=5$	$w=6$
1	29	1	N A	N A	N A	N A	N A	N A
1	29	2	0	0	0.664	0.664	0.664	0.664
1	29	3	0	1.580	1.580	1.580	1.580	1.580
1	29	4	0	0	0	0	1.600	1.600

Using the most cost-efficient tactic in each weatherband  $w$  gives the following effectiveness values  $E_{i,j,w}$  :

**Table 9. EFFECTIVENESS OF THE NEXT FEASIBLE COST - EFFICIENT TACTIC:** In each weatherband  $w$  the first applicable effectiveness value in Table 8 greater than zero is selected.

	$w=1$	$w=2$	$w=3$	$w=4$	$w=5$	$w=6$
$E_{i,j,w}^*$	0.000	1.580	0.664	0.664	0.664	0.664

which results in the averaged effectiveness:

$$\bar{E}_{i,j} = \sum_w PR_w \times E_{i,j,w}^* = 0.682 .$$

Note that the effectiveness has increased on account of the lack of weapon type  $k=3$ ! The SELECTOR output is ordered according to cost-effectiveness (not effectiveness), so it is quite possible that tactics far down in the Preferred Weapon List may actually be quite effective. These tactics typically have high associated attrition, but attrition is not considered in HEAVY ATTACK once SELECTOR has done its job.

By considering the same logic, it can be observed that the fourth order tactic on the Preferred Weapon List with  $E_{i,j,r,w} = 1.600$  will never be used. This is because the third

order tactic uses the same weapon (in this case weapon type  $k = 17$ ) in at least the same worst weatherband as tactic  $r = 4$ .

#### D. TIME IN HEAVY ATTACK

Once the effectiveness values  $\bar{E}_{i,j}$  are evaluated, the required input data is available in order to optimize the number of sorties assigned to the different target types. For most cases all targets are not killed when the optimization is finished because of the constrained number of sorties in the RAND - model. As in a real war scenario, the outcome of a given attack will influence subsequent target consideration and planning. Only the targets that survived the previous attack will be reconsidered. Weapons are not resupplied and therefore may become exhausted. The current version of HEAVY ATTACK may actually allocate *more* weapons in a given period than are available at the beginning of the period. This is because there is no explicit constraint on weapon usage. The deletion is currently done after each period by computing weapon usage after the optimization for the period is finished. However, a weapon will be deleted in the next period if it is exhausted at the end of the current period.

There is no resupply of targets between periods in HEAVY ATTACK, although there is a facility for reconstituting targets that have already been killed. This will be discussed later. Aircraft are also not resupplied or even directly represented in HEAVY ATTACK: the number of sorties available during each period is a direct input. Each time period represents an attack which changes the input for the following time period.

The fact that the importance of a target will change with time is represented in HEAVY ATTACK by the option of changing the military worth for each target type at the beginning of a new time period. Even though the military worth of a target is known in all future periods, the current sequential time optimization only "sees" the worth of a target for the current time period. Following from this "myopic" way of maximizing the military worth of killed targets it may happen that sorties are assigned in a time period to a target type when its military worth is relatively low. A "global" (or overall) time optimization can be expected to achieve a higher military worth of killed targets. This is discussed later.

## E. THE NONLINEAR MODEL IN HEAVY ATTACK

The basic structure of the current model in HEAVY ATTACK for one time period is given by:

### Parameter

$T_j$	number of type j targets available at the beginning of a time period
$D_j$	number of dead type j targets at the beginning of a time period
$V_j$	military worth of type j target during the current time period
$c_j$	target - parameter for type j target
$S_i$	number of type i sorties available for the current time period
$PROP_i$	proportion of $S_i$ that can be assigned

### Variables

$SX_{i,j}$	number of type i sorties that are assigned to type j targets
$KILL_j$	number of type j targets killed in the current time period

## Model

$$\text{Max } z = \sum_j V_j \times KILL_j$$

s.t.

$$KILL_j = f\left(T_j, c_j, D_j, \sum_i \bar{E}_{i,j} \times SX_{i,j}\right) \quad \forall j$$

where:

$$f\left(T_j, c_j, D_j, \sum_i \bar{E}_{i,j} \times SX_{i,j}\right) = \left(\frac{T_j}{c_j} - D_j\right) \times \left(1 - e^{-\frac{c_j}{T_j} \times \sum_i \bar{E}_{i,j} \times SX_{i,j}}\right)$$

The above function is the same function as used by RAND [Ref. 2].

$$\sum_j SX_{i,j} \leq PROP_i \times S_i \quad \forall i$$

$$0 \leq KILL_j \leq T_j - D_j \quad \forall j$$

$$0 \leq SX_{i,j} \quad \forall i, j$$

The nonlinear function  $f(T_j, c_j, D_j, \sum_i \bar{E}_{i,j} \times SX_{i,j})$  is of the same form as in the RAND - model. The number of targets of type  $j$  that are killed and the number of sorties of type  $i$  are constrained. The consumption of weapons is not considered in the model itself. After the optimal numbers of sorties are determined by the optimization, the consumption of the different weapon types is evaluated by:



$$\{ \text{consumption of weapon} \}_k = \sum_i \sum_j SX_{ij} \times \left( \sum_w PR_w \times B_{i,j,w}^* \right)$$

where the sum is over all  $\{ i, j, w \}$  such that  $k = K_{i,j,w}$ .

## F. TARGET RECONSTITUTION IN HEAVY ATTACK

The ability to reconstitute killed targets is a common fact in a modern war. HEAVY ATTACK records the number and type of targets as well as the time period when they are destroyed. After each optimization, it determines if targets can be reconstituted and evaluates the maximal number that are possible. A major task in this Thesis has been to determine the conditions under which reconstitution is allowed to happen by analyzing the responsible part of the HEAVY ATTACK source code. HEAVY ATTACK's logic seems to be as outlined below:

### Definition of index

$j$	target type index	$\forall j$
$p, pp$	time period index	$\forall p, pp \in \{1, 2, \dots, n\}$

### Parameter

$TIME_p$	length of time period $p$ in days	$\forall p$
$RECON_j$	minimum number of days a target has to stay dead	$\forall j$
$QTY_j$	maximum number of targets $j$ that can be reconstituted in 30 days	$\forall j$

### Aggregated parameter

$PERUP_{j,p}$	index of the last time period considered for reconstitution.
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If a target of type  $j$  is killed in time period  $PERUP_{j,p}$  or earlier, then there is sufficient time available to reconstitute the target so that it once again will be available in period  $p+1$ . The parameters  $TIME_p$  and  $RECON_j$  determine  $PERUP_j$  according to the following formula in HEAVY ATTACK:

Let

$$k_{j,\bar{p},p} = \begin{cases} 1 & \text{if } RECON_j < \sum_{p'=\bar{p}}^{p+1} TIME_{p'} - CEIL(0.5 \times TIME_{\bar{p}}) \\ 0 & \text{otherwise} \end{cases} \quad \forall j, \bar{p} \leq p < n$$

where the function  $CEIL$  rounds a real number to the next higher integer value.

$k_{j,\bar{p},p}$  indicates whether targets killed in period  $\bar{p}$  are eligible for reconstitution in period  $p$  and therefore:

$$PERUP_{j,p} = \sum_{\bar{p}=1}^p k_{j,\bar{p},p} \quad \forall j, p < n$$

Note that always  $PERUP_{j,p} \leq p$ .

## Variables

$KILL_{j,p}$  number of targets type  $j$  killed in time period  $p$   $\forall j, p$

$REBUILD_{j,p}$  maximum number of targets of type  $j$  that are reconstituted  
as live targets in time period  $p+1$   $\forall j, p < n$

## Conditions for Reconstitution

A killed target of type  $j$  can be reconstituted if the following 4 conditions are true:

1. at least a fraction of target  $j$  was destroyed in the previous or the current time period  $p$ ,
2. it has been dead for more than some defined time,

3. the total number of targets being reconstituted has to be less than the total number of targets which exceeds the minimum dead time

$$\sum_{p'=1}^p REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \quad \forall j, p < n$$

4. the maximum number of targets type  $j$  which can be reconstituted at the end of each time period  $p$  is given by:

$$REBUILD_{j,p} \leq \frac{QTY_j}{30} \times TIME_{p+1} \quad \forall j, p < n$$

where  $\frac{QTY_j}{30}$  represents the reconstitution rate per day.

This leads to the following submodel:

$$\max \quad z = \sum_j \sum_p REBUILD_{j,p}$$

s.t.

$$\sum_{p'=1}^p REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \quad \forall j, p < n \quad (A)$$

$$REBUILD_{j,p} \leq \frac{QTY_j}{30} \times TIME_{p+1} \quad \forall j, p < n \quad (B)$$

The interpretation of (A) is that the number of targets of type  $j$  rebuilt in period  $p$  or before cannot exceed the total number of targets that are killed during or before period  $PERUP_{j,p}$ . The interpretation of (B) is that the number of targets of type  $j$  rebuilt in period  $p$  cannot exceed a certain quantity depending on the length of period  $p$  and on the target type. There are no targets reconstituted in the last time period  $p=n$ .

### III. BOUNDS ON WEAPON CONSUMPTION

#### A. INTRODUCTION OF A WEAPON CONSTRAINT

A desired improvement for the current HEAVY ATTACK model is to add an additional constraint on the utilization of weapons inside the RAND - model.

Two important facts should be recalled:

1. For each sortie - target combination  $\{ i, j \}$  and each weatherband there is at most one weapon which can be used.
2. Averaging over all weatherbands is related to the probability that weatherband  $w$  might occur at the time sortie type  $i$  is assigned to target type  $j$ .

Let the upper bound on weapon consumption be defined as:

$WP_k$  total number of weapons of type  $k$  available

The required constraint for the consumption on weapons is then:

$$\sum_i \sum_j SX_{i,j} \times \left( \sum_w PR_w \times B_{i,j,w}^* \right) \leq WP_k \quad \forall k$$

where the sum is over all  $\{ i, j, w \}$  such that  $k = K_{i,j,w}$

#### B. REVISED MODEL OF HEAVY ATTACK

Reconstitution can be included in the RAND - model. Instead of considering reconstitution as a computational "bookkeeping" process, it can be part of the optimization. To accomplish this, it is necessary to define a new variable for the number of dead targets such that the time period as an additional dimension is represented by a second subscript:

$D_{j,p}$  is the total number of targets of type  $j$  killed in time periods  $< p$  less the number of targets that are reconstituted during this time :

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \quad \forall j, p$$

The military worth of a target is also time dependent:

$V_{j,p}$  military worth of a target type  $j$  in time period  $p$

**Embellished Thesis Model** (solved sequentially for  $p = 1, 2, 3, \dots, n$ )

$$\text{Max } z_p = \sum_j (V_{j,p} \times KILL_{j,p})$$

s.t.

$$KILL_{j,p} = f \left\{ T_j, c_j, D_{j,p}, \sum_i SX_{i,j} \times \left( \sum_w PR_w \times E_{i,j,w}^* \right) \right\} \quad \forall j$$

where :  $f\{ \dots \}$  is one of three functions discussed in the next chapter.

$$KILL_{j,p} \leq T_j - D_{j,p} \quad \forall j$$

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \quad \forall j$$

$$\sum_{p'=1}^p REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \quad \forall j$$

$$\sum_j SX_{i,j} \leq PROP_i \times S_i \quad \forall i$$

$$\sum_i \sum_j \left\{ SX_{i,j} \times \left( \sum_w PR_w \times B_{i,j,w}^* \right) \right\} \leq WP_k \quad \forall k$$

where the sum is over all  $\{ i, j, w \}$  such that  $k = K_{i,j,w}^*$

$$0 \leq SX_{i,j} \quad \forall i, j$$

$$0 \leq KILL_{j,p} \quad \forall j$$

$$0 \leq D_{j,p} \quad \forall j$$

$$0 \leq REBUILD_{j,p} \quad \forall j$$

where the upper bound on  $REBUILD_{j,p}$  is such that:

$$REBUILD_{j,p} \begin{cases} \leq \frac{QTY_j}{30} \times TIME_{p+1} & \text{if } p < n \\ = 0 & \text{if } p = n \end{cases} \quad \begin{matrix} \forall j \\ \forall j \end{matrix}$$

The model was written in the *General Algebraic Modeling System (GAMS)* [Ref. 3]. All optimization problems throughout the Thesis are solved with the nonlinear programming solver *MINOS - Version 5.0* [Ref. 4]. A database for 2 sortie-, 26 target- and 29 weapon-types was provided [Ref. 5] in order to compare the results by using three different objective functions, each over four time periods.

#### IV. LINEAR VERSUS NONLINEAR MODEL

In this chapter the derivation of the nonlinear objective function used by RAND is given. In addition two alternatives are represented by introducing the Washburn-Equation and the linear case in which the number of killed targets is proportional to the number of assigned sorties. Each of the three objective functions is used in the model described in the previous chapter for sequentially optimizing sortie assignments over four time periods. In order to compare the effect of the three objective functions, a measurement for the diversity of the allocated kill capability is defined.

##### A. RAND EQUATION

If  $K_j$  represents the total number of killed targets of type  $j$  then the objective function used in the RAND - model can be derived from the differential equation:

$$\frac{d K_j}{d X_j} = 1 - c_j \times \frac{K_j}{T_j} \quad (A)$$

$$\text{where } X_j = \sum_i \bar{E}_{ij} \times S X_{ij} \text{ and } 0 \leq c_j \leq 1$$

The differential equation (A) with the initial condition  $K_j(X_j = 0) = D_j$  has the solution:

$$K_j = \frac{T_j}{c_j} \times \left\{ 1 - \left( 1 - c_j \times \frac{D_j}{T_j} \right) \times e^{-\frac{c_j}{T_j} \times X_j} \right\}$$

Instead of bounding  $K_j$  by

$$D_j \leq K_j \leq T_j$$

let  $KILL_j$  be the number of targets killed in excess of  $D_j$  :

$$KILL_j = K_j - D_j$$

so that

$$0 \leq KILL_j \leq T_j - D_j$$

which leads to the final result:

$$KILL_j = \left( \frac{T_j}{c_j} - D_j \right) \times \left( 1 - e^{-\frac{c_j}{T_j} \times X_j} \right)$$

## B. LINEAR EQUATION

A special case for the differential equation (A) appears when  $c_j = 0$ :

then

$$\frac{d K_j}{d X_j} = 1$$

which yields:

$$K_j = X_j + D_j$$

so that

$$D_j \leq K_j \leq T_j$$

or by using

$$KILL_j = K_j - D_j$$

so that

$$0 \leq KILL_j \leq T_j - D_j$$

where the final solution represents the linear case:

$$KILL_j = X_j$$



Figure 1 illustrates the influence of the target parameter  $c_j$  on the function  $KILL_j = f(X_j)$ .

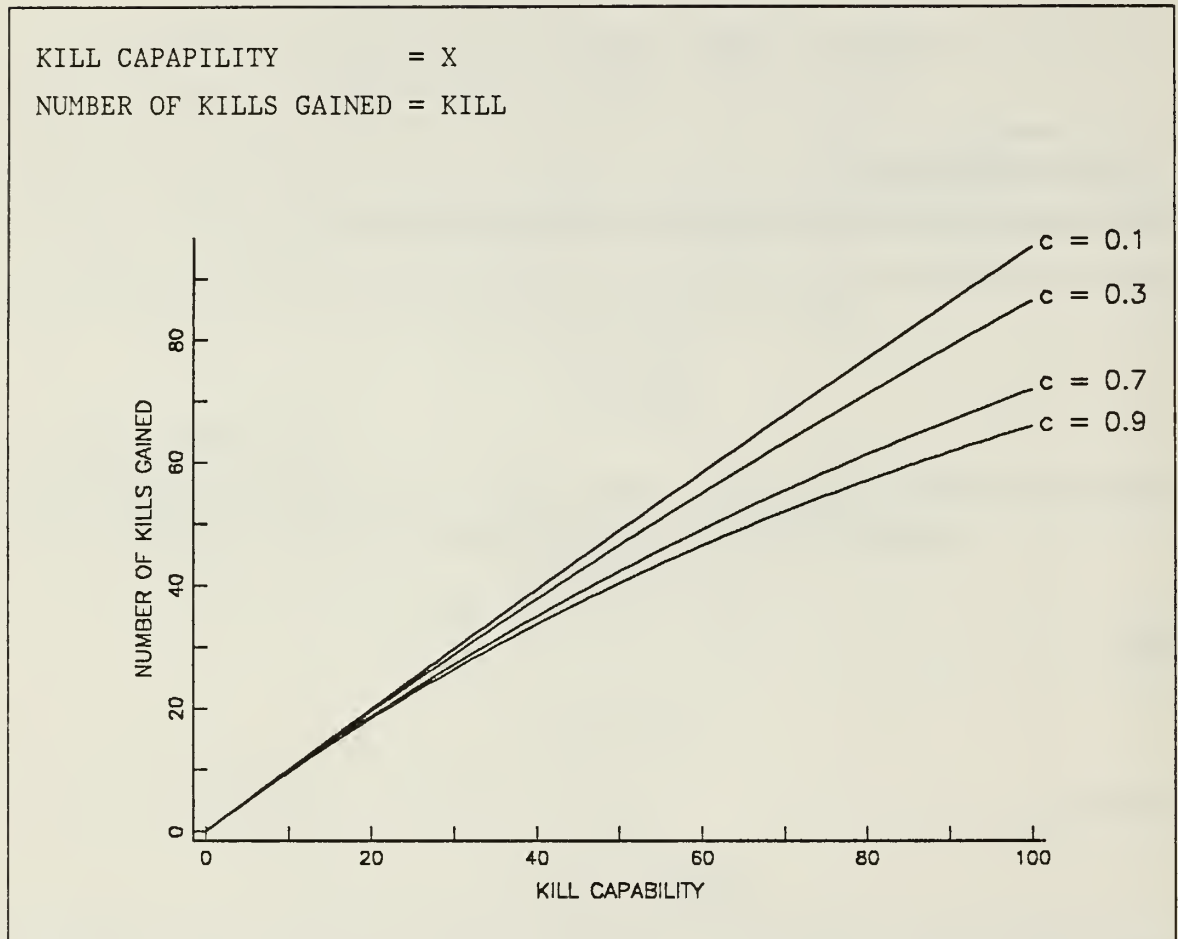


Figure 1. Influence of the target parameter  $c$  on the RAND-Equation: The solution of the differential equation used in the RAND-model is graphically shown for 4 different target parameters  $c$ .

The parameter  $c_j$  has no direct physical motivation. The model considered in the next section also contains a single parameter, but the parameter can be motivated physically.

### C. WASHBURN EQUATION

The Washburn - Equation [Ref. 6: p. 25] defines the differential  $\frac{d K_j}{d X_j}$  in the following way:

$$\frac{d K_j}{d X_j} = \text{Probability \{ attacking a live target \}}$$

or equivalently:

$$\frac{d K_j}{d X_j} = \frac{\{ \text{number of live targets} \}}{\{ \text{number of targets that look alive} \}}$$

This leads to the differential equation:

$$\frac{d K_j}{d X_j} = \frac{T_j - K_j}{T_j - K_j + \alpha_j \times K_j} \quad (B)$$

where  $\alpha_j$  is a constant proportion of killed targets, which have the property to appear live to a potential attacker.

The differential equation (B) with the initial condition  $K_j(X_j = 0) = D_j$  has the solution:

$$K_j = T_j \times \left\{ 1 - \left( 1 - \frac{D_j}{T_j} \right) \times e^{\frac{(1 - \alpha_j) \times (K_j - D_j) - X_j}{\alpha_j \times T_j}} \right\}.$$

Using  $KILL_j$  instead of  $K_j$  such that:

$$KILL_j = K_j - D_j$$

leads to the implicit solution for the Washburn - Equation as:

$$KILL_j = (T_j - D_j) \times \left( 1 - e^{\frac{(1 - \alpha_j) \times KILL_j - X_j}{\alpha_j \times T_j}} \right).$$

The difference between the two differential equations (A) and (B) for two different target parameters is shown in Figure 2 on page 24 . Observe that for target parameter  $c$  close to 0 or 1 the Washburn-equation tends to behave similarly to the RAND-equation.

Target parameter  $\alpha$  is denoted in the figure by  $c$ .

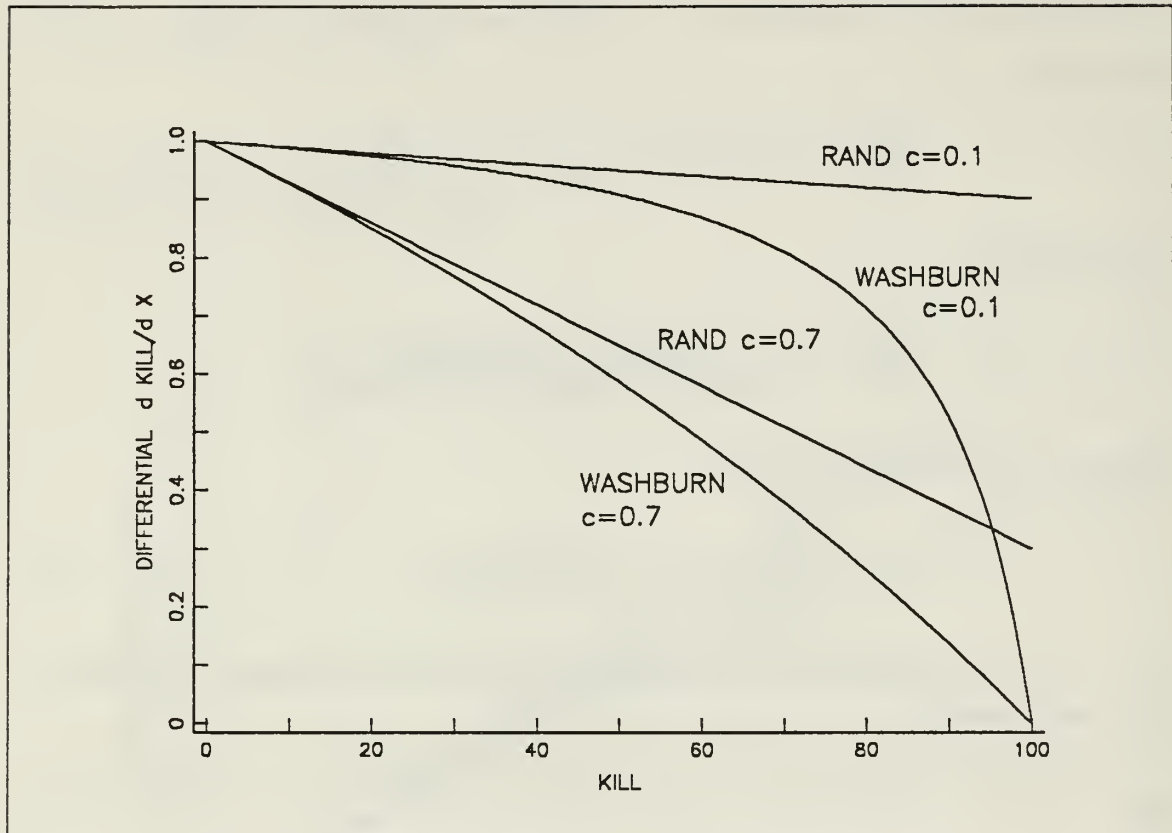


Figure 2. RAND- and Washburn-Diff. Equation with varied parameter  $c$ : The two differential equations are shown for 2 different target parameters  $c$ . Because the solution of the Washburn-Equation can be given only in an implicit form, the differential equations are shown rather than their solutions.

The influence of the three different objective functions on the RAND-model using the same input data is shown in Figure 3.

The total worth of killed targets decreases with time for each objective function. The main reason for this is that in the first time period sorties are assigned to those target types for which the effectiveness is highest. When all targets are killed, sorties are then assigned in the following time periods to the remaining targets for which the effectiveness is less. As a result, more and more sorties need to be allocated in order to gain the same number of killed targets. The number of reconstituted targets available at the beginning of the second or third period is relatively small or even zero and can therefore be neglected at this point. Since the variation in the number of sorties and in the mag-

nitude of the target values is too small to compensate for this effect, a declining trend in the objective function value over time for all three cases is observed.

Note that the Washburn-Equation always yields a smaller value than the RAND-Equation. This follows from the fact that the Washburn-Equation declines faster than the RAND-Equation for the same target parameter  $c$  as shown in Figure 2. The linear equation is larger than either one. The most important difference is not in the absolute level of target value killed, but rather in the influence of the objective function on the distribution of sorties over targets. This subject is taken up in the next section.

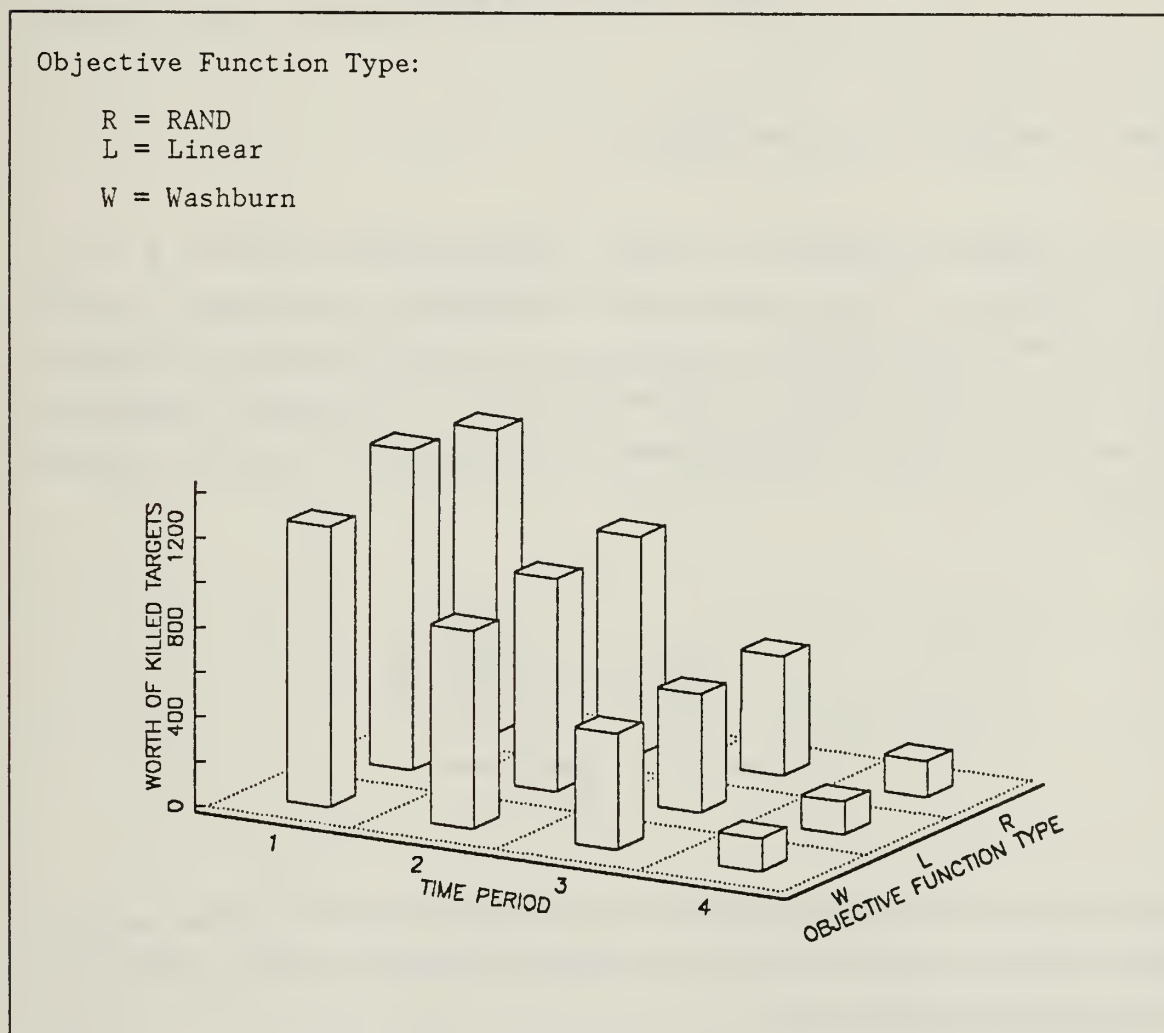


Figure 3. Total Military Worth of Killed Targets: represented for each different objective function and each time period by the height of the respective block in the figure.

#### D. DIVERSITY OF KILLED TARGETS

An important reason for USAF to use a nonlinear objective function is to avoid an undesired concentration of attacking sorties on a few targets. In analysing the effect of the three different objective functions on the optimization, a measurement is needed in order to indicate how many of the allocated sorties are spread over different targets.

In information theory the function

$$h(\mathbf{p}) = \sum_i \left( p_i \times \log \frac{1}{p_i} \right)$$

where  $\mathbf{p} = (p_1, p_2, \dots, p_n)$  and  $\sum_i p_i = 1$

is used to express the diversity or "entropy" of the probability distribution  $\mathbf{p} = \{p_i\}$ . Observe that  $h(\mathbf{p}) = 0$  when  $\mathbf{p}$  concentrates all probability in one element. The maximum possible value when  $\mathbf{p}$  has  $n$  elements occurs when they are all equal, in which case  $h(\mathbf{p}) = \log n$ . The diversity  $h(\mathbf{x})$  of an arbitrary set  $\{x_j\}$  of nonnegative members can be measured by simply normalizing them so that they sum to 1 and then computing entropy:

$$h(\mathbf{x}) = \frac{\sum_j x_j \times \log \left[ \frac{\sum_j x_j}{x_j} \right]}{\sum_j x_j}$$

The diversity of values  $h(\mathbf{x})$  gained from the same input data and model as used in the previous chapter is depicted in Figure 4. Since the number of targets  $n$  equals 26, the maximum diversity value will be

$$h(\mathbf{x})_{\max} = 3.26$$

Figure 4 makes it clear that the Linear objective function has a lower diversity value than the other two. This is to be expected, and in fact one of the main reasons for using

a nonlinear objective in the first place was to avoid low diversity values. However, note that:

1. The Linear diversity is not 0; that is, several target types are still attacked.
2. None of the objective functions achieves complete (3.26) diversity.

The differences emerge most strongly in period 3. Only 4 target types are attacked when the linear model is used, or 6 with the RAND-model. 16 different target types are attacked when the Washburn-equation is used; this is in keeping with the idea that the Washburn-equation is the most "non-linear" of the three (see Figure 2). The three models differ much less in period 1,2 or 4.

Objective Function Type:

R = RAND

L = Linear

W = Washburn

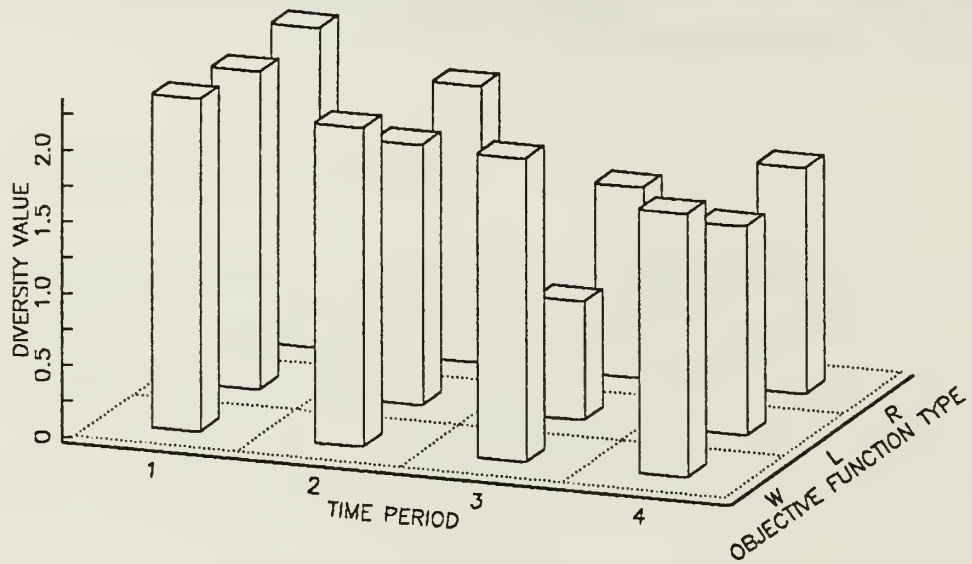


Figure 4. Diversity of killed targets for different objective functions: The height of each block illustrates to how many different target types (out of 26) sorties are allocated at different time periods by using each of the three objective functions.



## V. ALLOCATION OF SECONDARY WEAPONS

### A. COST-EFFICIENCY VERSUS KILL-EFFECTIVENESS

Cost considerations are finished once SELECTOR has established the Preferred Weapon List. Although this list contains different tactics, ordered in terms of cost-efficiency, HEAVY ATTACK only uses the top one on the list which is feasible. The only time at which HEAVY ATTACK may proceed to a succeeding tactic appears, as mentioned before, when a weapon has been exhausted in earlier periods.

As a second revision of HEAVY ATTACK, the model is changed to continue target attacks after the weapon type used by the most cost-effective tactic has been exhausted, using those weapons still on hand.

### B. A NONCONVEX CONSTRAINT

The model discussed in the previous chapter requires that only the tactic on the top of SELECTOR's Preferred Weapon List can be used. Once the corresponding weapon type is depleted further attacks by that sortie type in that weatherband against that target type are impossible. The idea in this section is to relax this strict requirement to permit using whatever tactic is highest on SELECTOR's list *among those whose weapons have not been exhausted*.

Implementing this logic in the existing model requires a modification of the variable  $SX_{i,j}$ :

$$SX_{i,j,r,w} = \text{number of sorties of type } i \text{ assigned to target of type } j \text{ which use} \\ \text{tactic } r \text{ in weatherband } w$$

The probability that all sorties of type  $i$  assigned to target of type  $j$  will attack the target in weatherband  $w$  has to be equal to the probability that weatherband  $w$  occurs at that time:

$$\sum_r SX_{i,j,r,w} = PR_w \times \sum_r \sum_{w'} SX_{i,j,r,w'}$$

Upon these redefined variables for the number of assigned sorties, it is possible to determine the utilization of each weapon type:

let  $WEAP_k$  be the consumption of all weapons of type  $k$

$$\text{then} \quad WEAP_k = \sum_i \sum_j \sum_r \sum_w (B_{ij,r,w} \times SX_{ij,r,w}) \quad \forall k$$

where the sum is over all  $\{i, j, r, w\}$  such that  $k = K_{ij,r,w}$ .

In order to assign sorties using less cost-effective tactics,  $SX_{ij,r,w}$  must be 0 unless the weapon types corresponding to all more cost-effective tactics are exhausted. The following constraint will enforce this logic:

$$0 = SX_{ij,r,w} \times \sum_{r'=1}^{r-1} (WP_k - WEAP_k) \quad \forall i, j, r, w \quad (C)$$

where  $k = K_{ij,r',w}$ .

The above constraint requires that at least one of the two factors on the right hand side of the equation equals zero, so either no sorties are assigned (first factor zero) or else all more cost-effective weapons are exhausted (second factor zero). The constraint thus enforces the desired logic, but there is a disadvantage in using it. The disadvantage is that the function on the right hand side of (C) is not only nonlinear (products of variables are involved) but nonconvex. Without constraint convexity, there is no guarantee that the locally optimal solutions achieved by the *MINOS* solver are globally optimal. There is some evidence, however, that globally optimal solutions are actually being attained. For one thing, employing constraint (C) always results in a higher objective function value than when only the most cost-efficient tactic is permitted. In addition, some experiments were performed where the improved model was changed into a linear model by linearizing the objective function at the optimal solution. The nonconvex constraint was then converted into a linear constraint by using integer variables. The optimal solution of this linearized model was identical to the solution gained by the nonlinear model with the nonconvex constraint.

### C. REVISED MODEL

The mathematical model is solved sequentially for  $p = 1, 2, \dots, n$ .

$$\text{Max } z_p = \sum_j (V_{j,p} \times KILL_{j,p})$$

s.t.

$$KILL_{j,p} = \left( \frac{T_j}{c_j} - D_{j,p} \right) \times \left( 1 - e^{-\frac{c_j}{T_j} \times X_j} \right) \quad \forall j$$

$$\text{where } X_j = \sum_i \sum_r \sum_w (E_{i,j,r,w} \times SX_{i,j,r,w})$$

$$KILL_{j,p} \leq T_j - D_{j,p} \quad \forall j$$

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \quad \forall j$$

$$\sum_{p'=1}^p REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \quad \forall j$$

$$\sum_j \sum_r \sum_w SX_{i,j,r,w} \leq PROP_i \times S_i \quad \forall i$$

$$WEAP_k = \sum_i \sum_j \sum_r \sum_w (B_{i,j,r,w} \times SX_{i,j,r,w}) \quad \forall k$$

where the sum is over all  $\{i, j, r, w\}$  such that  $k = K_{i,j,r,w}$

$$0 \leq SX_{lj,r,w} \times \sum_{r'=1}^{r-1} (WP_k - WLEAP_k) \quad \forall i, j, r, w$$

where  $k = K_{lj,r',w}$

$$\sum_r SX_{lj,r,w} = PR_w \times \sum_r \sum_{w'} SX_{lj,r,w'} \quad \forall i, j, w$$

$$0 \leq SX_{lj,r,w} \quad \forall i, j, r, w$$

$$0 \leq KILL_{j,p} \quad \forall j$$

$$0 \leq D_{j,p} \quad \forall j$$

$$0 \leq REBUILD_{j,p} \quad \forall j$$

where the upper bound on  $REBUILD_{j,p}$  is such that:

$$REBUILD_{j,p} \begin{cases} \leq \frac{QTY_j}{30} \times TIME_{p+1} & \text{if } p < n \\ = 0 & \text{if } p = n \end{cases} \quad \begin{matrix} \forall j \\ \forall j \end{matrix}$$

$$0 \leq WLEAP_k \leq WP_k \quad \forall k$$

The introduced relaxation will be used in the further revision of HEAVY ATTACK considered in the next chapter.

## VI. GLOBAL VERSUS MYOPIC TIME OPTIMIZATION

### A. TIME-DEPENDENT MILITARY WORTH OF TARGETS

When HEAVY ATTACK optimizes the allocation of sorties for each time period, it doesn't take advantage of the fact that the military worth of each target and each time period is known prior to running the optimization. The decision, which target type should be given a high priority to attack, is based on a comparison of military values of different target types restricted to the current time period. Although military worth of a target is given as a function of time, HEAVY ATTACK doesn't recognize the most favorable time for attacking a certain target type. This "myopic view" is caused by restricting the optimization to the time interval covered by one period.

It seems worthwhile to consider an optimization covering all time periods at once. This "global" optimization is expected to spend resources even more effectively than before, so that the total sum of gained military worth of killed targets might become higher compared to sequential time optimization. In addition, it can be expected that the number and type of killed targets in each time period will change.

The third revision for HEAVY ATTACK as presented in this chapter doesn't require major changes to the previously discussed model. A subscript for time is added to the variable  $SX_{i,j,r,w}$ :

$SX_{i,j,r,w,p}$       number of sorties of type  $i$  assigned to target type  $j$  by using tactic type  $r$  in weatherband  $w$  and in time period  $p$

The resources on sorties available needs to be defined as a function of sortie type and time:

$S_{i,p}$               maximum number of sorties type  $i$  available in period  $p$   
 $PROP_{i,p}$       proportion of  $S_{i,p}$  that can be assigned

Computing time increases with the number of time periods covered.

### B. GLOBAL MODEL

The mathematical model is shown below. The realization of this model in *GAMS*, including all inputs, is given in the Appendix.

$$\text{Max } z = \sum_j \sum_p (V_{j,p} \times KILL_{j,p})$$

s.t.

$$KILL_{j,p} = \left( \frac{T_j}{c_j} - D_{j,p} \right) \times \left( 1 - e^{-\frac{c_j}{T_j} \times X_{j,p}} \right) \quad \forall j, p$$

$$\text{where } X_{j,p} = \sum_i \sum_r \sum_w (E_{ij,r,w} \times SX_{ij,r,w,p})$$

$$KILL_{j,p} \leq T_j - D_{j,p} \quad \forall j, p$$

$$D_{j,p} = \sum_{p'=1}^{p-1} (KILL_{j,p'} - REBUILD_{j,p'}) \quad \forall j, p$$

$$\sum_{p'=1}^p REBUILD_{j,p'} \leq \sum_{p'=1}^{PERUP_{j,p}} KILL_{j,p'} \quad \forall j, p$$

$$\sum_j \sum_r \sum_w SX_{ij,r,w,p} \leq PROP_{i,p} \times S_{i,p} \quad \forall i, p$$

$$WEAP_k = \sum_l \sum_j \sum_r \sum_w \left( B_{l,j,r,w} \times \sum_p SX_{ij,r,w,p} \right) \quad \forall k$$

where the sum is over all  $\{i, j, r, w\}$  such that  $k = K_{ij,r,w}$

$$0 = SX_{ij,r,w} \times \sum_{r'=1}^{r-1} (WP_k - WEAP_k) \quad \forall i, j, r, w$$

where  $k = K_{ij,r',w}$



$$\sum_r SX_{i,j,r,w,p} = PR_w \times \sum_r \sum_{w'} SX_{i,j,r,w',p} \quad \forall i, j, w, p$$

$$0 \leq SX_{i,j,r,w,p} \quad \forall i, j, r, w, p$$

$$0 \leq KILL_{j,p} \quad \forall j, p$$

$$0 \leq D_{j,p} \quad \forall j, p$$

$$0 \leq REBUILD_{j,p} \quad \forall j$$

where the upper bound on  $REBUILD_{j,p}$  is such that:

$$REBUILD_{j,p} \begin{cases} \leq \frac{QTY_j}{30} \times TIME_{p+1} & \text{if } p < n \\ = 0 & \text{if } p = n \end{cases} \quad \begin{matrix} \forall j \\ \forall j \end{matrix}$$

$$0 \leq WEAP_k \leq WP_k \quad \forall k$$

### C. RESULTS AND COMPARISONS

The above model was too large to be run in *GAMS* on available computer equipment at reasonable cost with the same size of input data used previously. Therefore the number of target types were reduced from 26 to 13. Other efforts were also made to decrease required computing time.

Table 10, Table 11 and Figure 5 compare the results of the global and myopic sequential optimizations. The global optimization achieves more target value killed; the percentage gain for the global approach is  $(1358.0 - 1123.0)/1123.0 = 20.9\%$ . Comparing the target values of target type 5 and 27 over all 4 periods shows that the highest target value occurs in period 3. The global optimization realizes this fact by destroying all available targets at that time. While both target types, especially target type 5, have a relatively high target value in the first time period, most of these targets are therefore killed by myopic optimization in the first period.



**Table 10. NUMBER OF KILLED TARGETS:** The table shows the number of killed targets achieved by sequential and global optimization as well as the respective target value for each time period.

	Time Period 1			Time Period 2		
Target Type	Target Value	Killed Targets		Target Value	Killed Targets	
		Myopic	Global		Myopic	Global
TG 5	10	17.3	0.5	14	1.2	1.1
TG 8	10	13.0	13.0	10	0.0	0.0
TG 10	4	0.0	0.0	7	0.0	0.0
TG 11	7	0.0	9.6	9	0.0	0.0
TG 12	7	0.0	0.0	12	0.0	0.0
TG 13	4	0.0	2.2	5	0.0	0.0
TG 14	20	2.0	2.0	15	0.0	0.0
TG 22	2	0.0	0.0	2	0.0	0.0
TG 24	2	0.0	0.0	7	0.1	0.0
TG 25	5	0.0	0.0	12	22.3	26.6
TG 27	4	19.1	0.0	7	1.9	0.0
TG 29	7	0.0	0.0	7	0.0	0.0
TG 34	5	8.6	0.0	5	9.4	0.0
	Time Period 3			Time Period 4		
Target Type	Target Value	Killed Targets		Target Value	Killed Targets	
		Myopic	Global		Myopic	Global
TG 5	18	1.0	18.0	1.0	1.0	2.0
TG 8	10	0.0	0.0	0.7	0.0	0.0
TG 10	10	5.4	0.0	3.1	23.6	26.3
TG 11	10	4.3	0.0	2.1	3.5	0.0
TG 12	18	0.0	0.0	2.1	0.0	0.0
TG 13	7	4.0	1.7	1.0	0.0	0.1
TG 14	10	0.	0.0	0.7	0.0	0.0
TG 22	2	0.0	0.0	2.0	6.0	6.0
TG 24	10	2.2	1.5	2.5	0.4	1.3
TG 25	10	5.5	1.1	0.9	0.0	0.0
TG 27	8	0.0	21.0	2.0	0.0	0.0
TG 29	8	0.0	0.0	1.0	0.0	0.0
TG 34	8	0.0	18.0	0.7	0.0	0.0

Table 11. **MILITARY WORTH OF KILLED TARGETS:** gained by sequential and by global optimization is given for each time period and as a total sum.

	Myopic Optimization	Global Optimization
Time Period 1	462.8	251.3
Time Period 2	345.5	333.8
Time Period 3	220.1	674.0
Time Period 4	94.6	98.9
Total Worth of Killed Targets	1123.0	1358.0

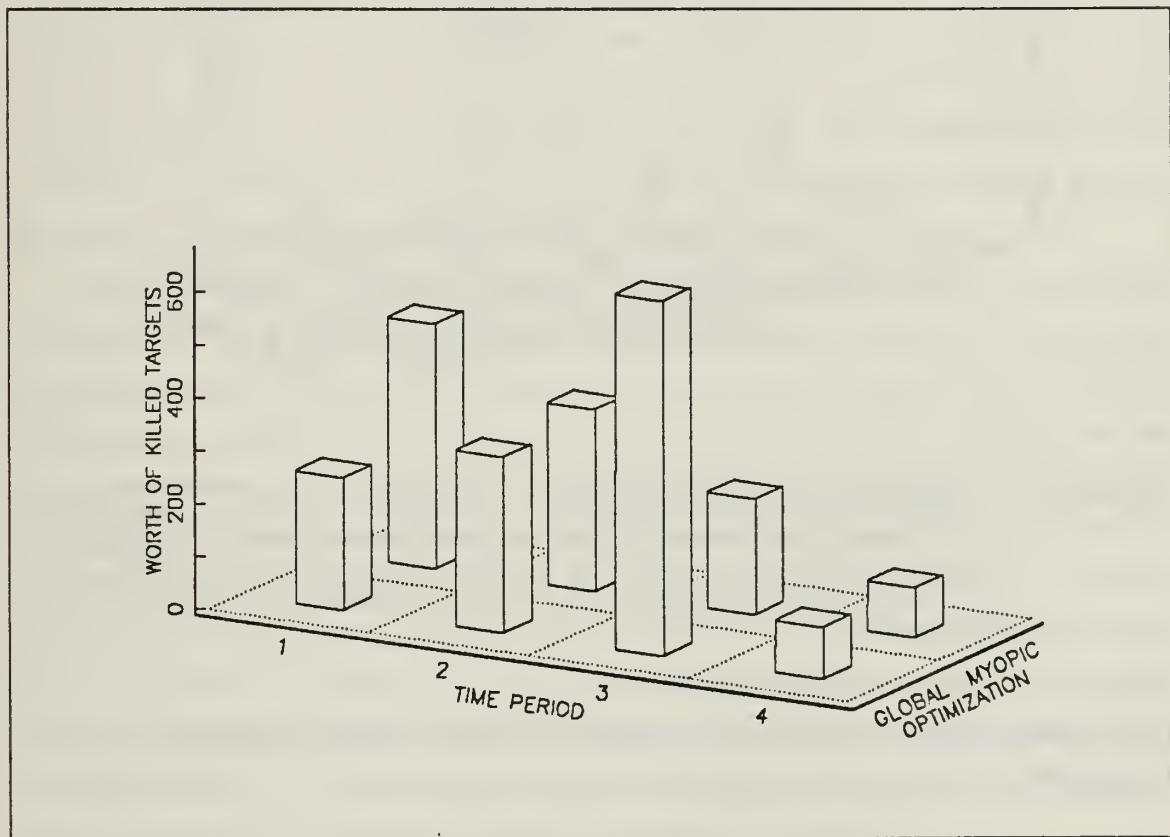
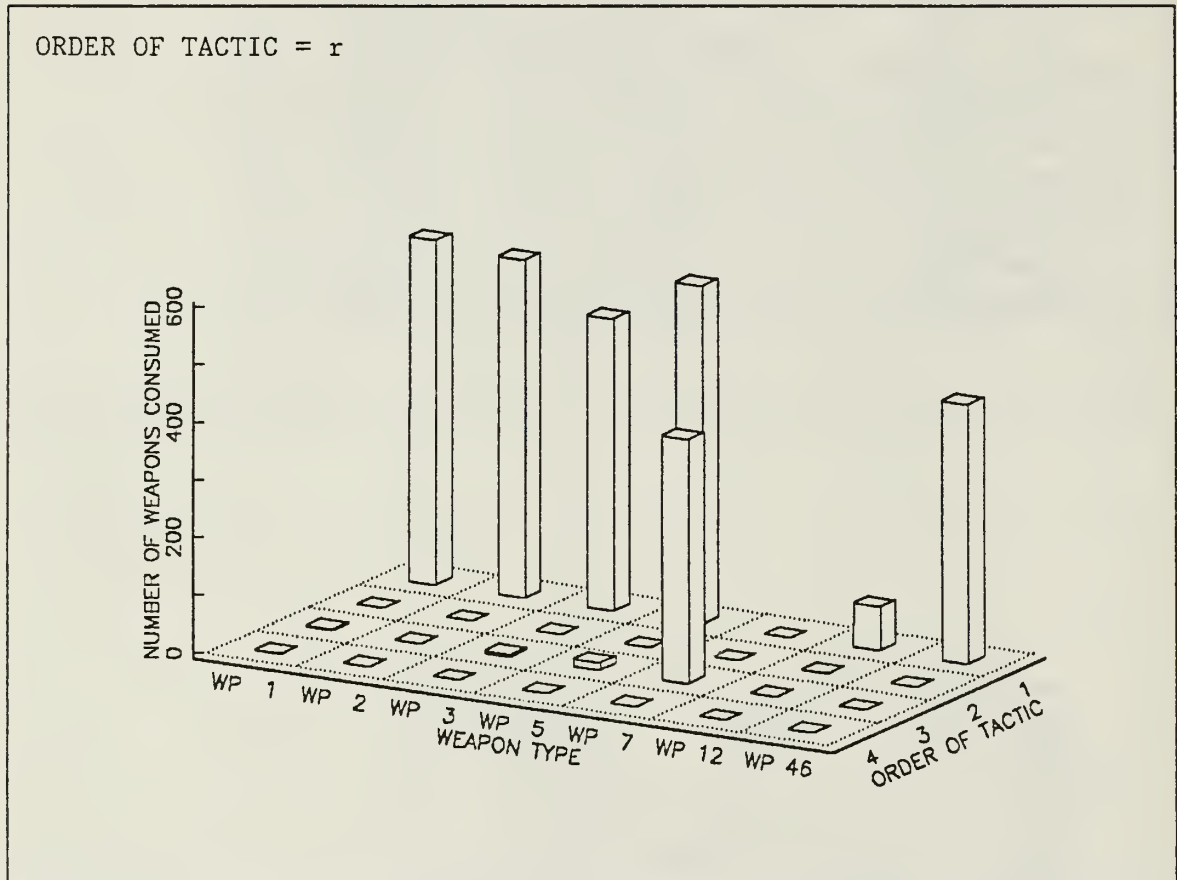


Figure 5. **Distribution of Military Worth of Killed Targets:** The height of each block represents the numerical value given in Table 11 depending on the time period and on the kind of optimization used.

Both the global and the myopic models utilize secondary weapons. Figure 6 shows weapon usage in the global model. Note that weapon type WP7 is used extensively in situations where more cost-effective weapons are exhausted.



**Figure 6. Allocation of Secondary Weapons:** The height of each block represents the number of weapons utilized by the global optimization. A significant number of weapon type WP7 is used by tactics of order  $r=3$ . This is only possible when weapons used by tactics of order  $r=1$  and  $r=2$  are exhausted.

A more detailed report of the solution is given in the SOLVE SUMMARY of *GAMS* in the Appendix.

## VII. CONCLUSIONS

In the first revision of the current HEAVY ATTACK model, a weapon constraint is added and three different objective functions are compared. The objective function best used in the model depends on the priorities of the user:

1. Using a linear objective function instead of a nonlinear one has the advantage of simplicity and consequent computational efficiency. A disadvantage is a less dispersed allocation of sorties to different targets.
2. Using the Washburn - Equation instead of the RAND - Equation has the advantage of using a well defined target parameter. The dispersion of attacked target types might be somewhat less influenced due to changes in the input data.

In the second revision the current philosophy of using the most cost-efficient tactic is relaxed such that less cost-efficient tactics can be utilized within a time period. With this revision, tactics not at the top of the Preferred Weapon List (SELECTOR output) can be utilized if all more cost-effective tactics are infeasible due to weapon exhaustion. This revision is particularly important when there is a small number of time periods, since the same capability already exists *between* time periods.

The third revision replaces sequential optimization (current practice) with global optimization. The comparison between sequential and global optimization by using the same input data shows a qualitative difference in the achieved results. There is a definite indication that sequential time optimization tends to achieve military success in the beginning of the war by sacrificing the potential for later success. Global optimization tends to husband weapons and even targets (in cases where target value increases with time) for later periods in the war. An argument for global optimization can be based on the fact that it is more efficient in killing targets with large military values. On the other hand, it could also be argued that sequential optimization is more likely to imitate what will actually happen, "optimal" or not. In any case, if global optimization is used, then the distribution of the value of destroyed targets seems to be much more time dependent than is recognized by the current method of sequential optimization.

All revisions introduced in this Thesis result in gaining of more military worth. USAF's general objective is to determine their future need of weapons rather than to maximize the military worth of killed targets. With the revisions described above, utilization of weapons plays a more important and direct role in the optimization, especially

when more than one tactic is considered. The developed models are intended to provide the necessary structure to embellish HEAVY ATTACK for this purpose.

## APPENDIX    GLOBAL OPTIMIZATION MODEL

```

4
5 *****
6 *
7 * Math.Model: Klaus Wirths                      February 1989 *
8 *
9 * File Name : P H C R      G A M S                *
10 *
11 *
12 * Remark      : This Model is an improved version of the HEAVY ATTACK *
13 *              model; it contains a subset of a larger database.      *
14 *
15 * Specification:  RAND - Equation                    *
16 *                Multi-Weapon Optimization            *
17 *                Multi-Time Period (Global) Optimization *
18 *
19 *
20 * Reference : Dennis M. Coulter, Maj, USAF          *
21 *           War, Mobilization & Munitions Division *
22 *           Directorate of Plans, DCS/P&Q           *
23 *
24 *
25 *           Sortie Allocation by a Nonlinear Programming Model *
26 *           for Determining a Munitions Mix          *
27 *           R.J.Clasen, G.W.Graves and J.Y.Lu        *
28 *           RAND, Santa Monica                      March 1974 *
29 *
30 *****
31
32 SET
33     I      aircraft type index      / AC1 * AC2 /
34
35     J      target type index        / TG5
36                                     TG8
37                                     TG10
38                                     TG11
39                                     TG12
40                                     TG13
41                                     TG14
42                                     TG22
43                                     TG24
44                                     TG25
45                                     TG27
46                                     TG29
47                                     TG34 /
48
49     K      weapon type index        / WP1
50                                     WP2
51                                     WP3
52                                     WP4

```



```

53                                     WP5
54                                     WP6
55                                     WP7
56                                     WP8
57                                     WP9
58                                     WP10
59                                     WP11
60                                     WP12
61                                     WP15
62                                     WP18
63                                     WP19
64                                     WP21
65                                     WP22
66                                     WP24
67                                     WP25
68                                     WP27
69                                     WP34
70                                     WP42
71                                     WP45
72                                     WP46 /
73
74      W      weatherband type index / WB1 * WB6 /
75
76      R      order of preferred weapon type / OD1 * OD4 /
77
78      P      time period index      / PER1 * PER4 /           ;
79
80
81  ALIAS (J,JP)                                           ;
82
83  ALIAS (R,RP)                                           ;
84
85  ALIAS (P,PP)                                           ;
86
87  ALIAS (P,PPP)                                          ;
88
89  ALIAS (W,WPP)                                          ;
90
91
92  ***  Definition of TARGET Parameters
93
94  PARAMETERS
95
96      T(J)  total number of target type J
97
98  ****  all entries for T(J) has to be nonzero values  ****
99
100                                     / TG5      18
101                                     TG8      13
102                                     TG10     29
103                                     TG11     32
104                                     TG12      3
105                                     TG13      4
106                                     TG14      2
107                                     TG22      6
108                                     TG24      3

```



```

109                                     TG25  51
110                                     TG27  21
111                                     TG29   9
112                                     TG34  18  /
113
114
115
116      C(J)  TARGET parameter                                0 < C < 1
117
118                                     / TG5    0.2
119                                     TG8    0.1
120                                     TG10   0.2
121                                     TG11   0.1
122                                     TG12   0.1
123                                     TG13   0.3
124                                     TG14   0.1
125                                     TG22   0.2
126                                     TG24   0.8
127                                     TG25   0.3
128                                     TG27   0.7
129                                     TG29   0.1
130                                     TG34   0.2  / ;
131
132
133      TABLE  V(J,P)  value of target type J
134
135                                     PER1  PER2  PER3  PER4
136      TG5          10    14    18    1.0
137      TG8          10    10    10    0.7
138      TG10         4     7    10    3.1
139      TG11         7     9    10    2.1
140      TG12         7    12    18    2.1
141      TG13         4     5     7    1.0
142      TG14        20    15    10    0.7
143      TG22         2     2     2    2.0
144      TG24         2     7    10    2.5
145      TG25         5    12    10    0.9
146      TG27         4     7     8    2.0
147      TG29         7     7     8    1.0
148      TG34         5     5     8    0.7
149
150
151
152  **  Definition of Sortie numbers
153
154      TABLE  S(I,P)  maximum number of sorties for AC type I
155
156                                     PER1  PER2  PER3  PER4
157      AC1      180    200    150    300
158      AC2      180    200    150    300
159
160
161      TABLE  PROP(I,P)  proportion of available number of sorties for AC I
162
163                                     PER1  PER2  PER3  PER4
164      AC1      0.60   0.50   0.70   0.70

```

```

165          AC2    0.45    0.60    0.70    0.70
166
167
168 PARAMETER
169
170 **  Definition of WP numbers
171
172          WP(K)    maximum number of WP k - 100000 represents infinity
173
174                      / WP1      600
175                      WP2      100000
176                      WP3      100000
177                      WP4      100000
178                      WP5      600
179                      WP6      100000
180                      WP7      100000
181                      WP8      100000
182                      WP9      100000
183                      WP10     100000
184                      WP11     100000
185                      WP12     600
186                      WP15     100000
187                      WP18     100000
188                      WP19     100000
189                      WP21     100000
190                      WP22     100000
191                      WP24     100000
192                      WP25     100000
193                      WP27     100000
194                      WP34     100000
195                      WP42     100000
196                      WP45     100000
197                      WP46      450    /
198
199
200 **  Definition of Weatherband Distribution
201
202          PR(W)    probability of weatherband W
203          /  WB1    0.00
204             WB2    0.02
205             WB3    0.14
206             WB4    0.07
207             WB5    0.07
208             WB6    0.70    /
209
210
211 **  Parameter definition for Reconstitution
212
213          TIME(P)  length of time period P
214          /  PER1   3
215             PER2   4
216             PER3   8
217             PER4  15    /
218
219
220          RECON(J)  number of days a killed target has to stay dead

```

```

221
222          / TG5      3
223          TG8      35
224          TG10     20
225          TG11      7
226          TG12     35
227          TG13     37
228          TG14     40
229          TG22     32
230          TG24     30
231          TG25      8
232          TG27     30
233          TG29     40
234          TG34     34      /
235
236
237      QTY(J)      maximum number of targets to be reconst. in 30 days
238
239          / TG5      4
240          TG8      2
241          TG10     10
242          TG11      2
243          TG12      2
244          TG13      2
245          TG14      0
246          TG22      7
247          TG24      2
248          TG25     20
249          TG27      0
250          TG29      0
251          TG34      3      /
252
253
254      PERUP(J,P) upper bound on time periods considered for reconstitution;
255
256  * a killed target must exceed a minimum time > RECON(J) < before it
257  * is allowed to be reconstituted
258
259
260      LOOP((J,P),
261
262      PERUP(J,P) = SUM(PP$(ORD(PP) LE ORD(P)),1$(RECON(J) LT (SUM(PPP$
263      ( (ORD(PPP) LE (ORD(P)+1)) AND (ORD(PPP) GE ORD(PP)) ),TIME(PPP))
264      - CEIL(0.5 * TIME(PP)) ) ) )      ) ;
265
266
267
268
269
270  *****
271  *                                           *
272  *           Begin of aggregated INPUT DATA           *
273  *                                           *
274  *****
275
276

```

277	TABLE E(I,J,R)	Number of Targets type J killed by one Sortie type I			
278					
279					
280		OD1	OD2	OD3	OD4
281	AC1. TG5	.159	.156	.193	.310
282	AC1. TG8	.305	.418	.299	.327
283	AC1. TG10	.083	.120	.076	.276
284	AC1. TG11	.081	.092	.077	.034
285	AC1. TG12	.028	.010	.020	.044
286	AC1. TG13	.216	.269	.205	.208
287	AC1. TG14	.386	.328	.284	.292
288	AC1. TG22	.343	.468	.333	.305
289	AC1. TG24	.273	.232	.273	.218
290	AC1. TG25	.134	.072	.067	.042
291	AC1. TG27	.933	.913	.792	.741
292	AC1. TG29	.137	.139	.092	.117
293	AC1. TG34	.298	.172	.150	.428
294	AC2. TG5	.247	.241	.288	.282
295	AC2. TG8	.262	.305	.365	.418
296	AC2. TG10	.083	.120	.076	.276
297	AC2. TG11	.081	.092	.077	.034
298	AC2. TG12	.028	.010	.020	.044
299	AC2. TG13	.195	.216	.260	.269
300	AC2. TG14	.685	.552	.569	.388
301	AC2. TG22	.251	.343	.468	.350
302	AC2. TG24	.205	.206	.273	.138
303	AC2. TG25	.134	.072	.067	.042
304	AC2. TG27	.652	.933	.913	.792
305	AC2. TG29	.137	.064	.139	.092
306	AC2. TG34	.382	.367	.338	.231

307							
308							
309							
310	TABLE B(I,J,R,W)	Weaponload Array for each set <i j r w>					
311							
312		OD1.WB1	OD1.WB2	OD1.WB3	OD1.WB4	OD1.WB5	OD1.WB6
313	AC1. TG5	0	2	2	2	2	2
314	AC1. TG8	0	2	2	2	2	2
315	AC1. TG10	0	2	2	2	2	2
316	AC1. TG11	0	2	2	2	2	2
317	AC1. TG12	0	0	0	2	2	2
318	AC1. TG13	0	2	2	2	2	2
319	AC1. TG14	0	0	6	6	6	6
320	AC1. TG22	0	2	2	2	2	2
321	AC1. TG24	0	0	0	2	2	2
322	AC1. TG25	0	2	2	2	2	2
323	AC1. TG27	0	0	6	6	6	6
324	AC1. TG29	0	0	6	6	6	6
325	AC1. TG34	0	6	6	6	6	6
326	AC2. TG5	0	6	6	6	6	6
327	AC2. TG8	0	6	6	6	6	6
328	AC2. TG10	0	2	2	2	2	2
329	AC2. TG11	0	2	2	2	2	2
330	AC2. TG12	0	0	0	2	2	2
331	AC2. TG13	0	6	6	6	6	6
332	AC2. TG14	0	0	4	4	4	4

333	AC2.TG22	0	6	6	6	6	6
334	AC2.TG24	0	6	6	6	6	6
335	AC2.TG25	0	2	2	2	2	2
336	AC2.TG27	0	0	0	0	0	6
337	AC2.TG29	0	0	6	6	6	6
338	AC2.TG34	0	0	4	4	4	4
339							
340	+	OD2.WB1	OD2.WB2	OD2.WB3	OD2.WB4	OD2.WB5	OD2.WB6
341	AC1.TG5	0	2	2	2	2	2
342	AC1.TG8	0	0	0	0	0	0
343	AC1.TG10	0	0	0	0	0	0
344	AC1.TG11	0	0	0	0	0	0
345	AC1.TG12	0	0	0	0	0	0
346	AC1.TG13	0	0	0	0	0	0
347	AC1.TG14	0	0	0	0	0	0
348	AC1.TG22	0	0	0	0	0	0
349	AC1.TG24	0	0	6	6	6	6
350	AC1.TG25	0	0	0	0	0	0
351	AC1.TG27	0	0	0	0	0	0
352	AC1.TG29	0	0	0	0	0	0
353	AC1.TG34	0	0	0	2	2	2
354	AC2.TG5	0	6	6	6	6	6
355	AC2.TG8	0	2	2	2	2	2
356	AC2.TG10	0	0	0	0	0	0
357	AC2.TG11	0	0	0	0	0	0
358	AC2.TG12	0	0	0	0	0	0
359	AC2.TG13	0	2	2	2	2	2
360	AC2.TG14	0	4	0	0	0	0
361	AC2.TG22	0	2	2	2	2	2
362	AC2.TG24	0	0	0	0	0	0
363	AC2.TG25	0	0	0	0	0	0
364	AC2.TG27	0	0	6	6	6	6
365	AC2.TG29	0	0	0	6	6	6
366	AC2.TG34	0	0	0	6	6	6
367							
368	+	OD3.WB1	OD3.WB2	OD3.WB3	OD3.WB4	OD3.WB5	OD3.WB6
369	AC1.TG5	0	0	0	0	0	0
370	AC1.TG8	0	2	2	2	2	2
371	AC1.TG10	0	2	2	2	2	2
372	AC1.TG11	0	2	2	2	2	2
373	AC1.TG12	0	0	2	2	2	2
374	AC1.TG13	0	2	2	2	2	2
375	AC1.TG14	0	2	2	2	2	2
376	AC1.TG22	0	2	2	2	2	2
377	AC1.TG24	0	0	2	2	2	2
378	AC1.TG25	0	0	0	0	0	0
379	AC1.TG27	0	6	0	0	0	0
380	AC1.TG29	0	0	2	2	2	2
381	AC1.TG34	0	2	2	0	0	0
382	AC2.TG5	0	0	0	0	0	0
383	AC2.TG8	0	0	0	0	0	0
384	AC2.TG10	0	2	2	2	2	2
385	AC2.TG11	0	2	2	2	2	2
386	AC2.TG12	0	0	2	2	2	2
387	AC2.TG13	0	0	0	0	0	0
388	AC2.TG14	0	0	0	0	0	0

389	AC2. TG22	0	0	0	0	0	0
390	AC2. TG24	0	0	0	2	2	2
391	AC2. TG25	0	0	0	0	0	0
392	AC2. TG27	0	0	0	0	0	0
393	AC2. TG29	0	0	0	0	0	0
394	AC2. TG34	0	6	6	0	0	0
395							
396	+	OD4. WB1	OD4. WB2	OD4. WB3	OD4. WB4	OD4. WB5	OD4. WB6
397	AC1. TG5	0	6	6	6	6	6
398	AC1. TG8	0	6	6	6	6	6
399	AC1. TG10	0	0	2	2	2	2
400	AC1. TG11	0	0	0	0	0	0
401	AC1. TG12	0	0	0	0	0	0
402	AC1. TG13	0	6	6	6	6	6
403	AC1. TG14	0	6	0	0	0	0
404	AC1. TG22	0	6	6	6	6	6
405	AC1. TG24	0	0	0	0	0	0
406	AC1. TG25	0	0	0	0	0	0
407	AC1. TG27	0	0	2	2	2	2
408	AC1. TG29	0	0	0	0	0	0
409	AC1. TG34	0	0	0	0	0	0
410	AC2. TG5	0	0	0	0	0	0
411	AC2. TG8	0	0	0	0	0	0
412	AC2. TG10	0	0	2	2	2	2
413	AC2. TG11	0	0	0	0	0	0
414	AC2. TG12	0	0	0	0	0	0
415	AC2. TG13	0	0	0	0	0	0
416	AC2. TG14	0	6	6	6	6	6
417	AC2. TG22	0	0	0	0	0	0
418	AC2. TG24	0	0	0	0	0	0
419	AC2. TG25	0	0	0	0	0	0
420	AC2. TG27	0	6	0	0	0	0
421	AC2. TG29	0	0	2	2	2	2
422	AC2. TG34	0	0	0	0	0	0

TABLE WPTYPE(I,J,R)

\* For each sortie-target combination the weapon type K of order R  
 \* is given if it is possible to use this weapon

431		OD1	OD2	OD3	OD4
432	AC1. TG5	5	6	5	4
433	AC1. TG8	5	5	7	3
434	AC1. TG10	5	5	7	18
435	AC1. TG11	5	5	7	5
436	AC1. TG12	5	5	7	5
437	AC1. TG13	5	5	7	3
438	AC1. TG14	3	3	5	3
439	AC1. TG22	5	5	7	3
440	AC1. TG24	5	3	7	3
441	AC1. TG25	24	24	24	24
442	AC1. TG27	3	3	3	7
443	AC1. TG29	3	3	7	7
444	AC1. TG34	3	5	5	3



445	AC2.TG5	2	1	2	1
446	AC2.TG8	1	5	1	5
447	AC2.TG10	5	5	7	18
448	AC2.TG11	5	5	7	5
449	AC2.TG12	5	5	7	5
450	AC2.TG13	1	5	1	5
451	AC2.TG14	12	12	12	1
452	AC2.TG22	1	5	5	1
453	AC2.TG24	1	1	5	1
454	AC2.TG25	24	24	24	24
455	AC2.TG27	1	3	3	3
456	AC2.TG29	3	1	3	7
457	AC2.TG34	12	1	1	1
458					
459	*****				
460	*			*	
461	*	End of INPUT DATA		*	
462	*			*	
463	*****				
464					
465					
466					
467	** Definition of Sortie Variable				
468					
469	*	SX(I,J,R,W,P) describes the number of sorties type I assigned			
470	*	to a target of type J carrying any weapon feasible for tactic R			
471	*	and weatherband W and in time period P			
472					
473					
474	POSITIVE VARIABLES	SX(I,J,R,W,P)			;
475					
476	** Initial Values for Variables				
477					
478		SX.L(I,J,R,W,P) = 0			;
479					
480	** Declaration of variable EXPO(J,P)				
481					
482	POSITIVE VARIABLE	EXPO(J,P)			;
483					
484	** Declaration of Kill Variable				
485					
486	POSITIVE VARIABLE	KILL(J,P)			;
487					
488	** Declaration of Variable D(J,P)				
489					
490	POSITIVE VARIABLE	D(J,P)			;
491					
492	** Declaration of Variable for cumulative weapon consumption				
493					
494	POSITIVE VARIABLE	WEAP(K)			;
495					
496	** Upper bound for variable Weapon Consumption				
497					
498		WEAP.UP(K) = WP(K)			;
499					
500					



```

501  ** Declaration of variable for number of targets been reconstituted
502
503      POSITIVE VARIABLE REBUILD(J,P) ;
504
505  ** Upper bound for variable REBUILD
506
507      REBUILD.UP(J,P) = QTY(J) * TIME(P+1) / 30 ;
508
509
510
511  ** Variable definition for objective function
512
513      VARIABLE Z ;
514
515
516  EQUATIONS
517
518      KILLVAL                maximize the value of destroyed targets
519      KILLNL(J,P)            determines the number of killed targets
520      EXPONENT(J,P)          evaluates the values of the exponential terms
521      DEADTG(J,P)            determines the number of dead targets
522      KILLCON(J,P)            constraint the number of killed targets
523      RECCON(J,P)            constraint the max. number of targets for reconst.
524      SORTCON(I,P)           constraint the number of allocated sorties
525      WEAPCONSUM(K)           determines the consumption of each weapon type
526      SELECT(I,J,R,W)         decides if next weapon on list can be used
527      DISTR(I,J,W,P)          ensures that all weatherbands are covered prop. ;
528
529
530  KILLVAL..
531
532      Z =E= SUM((J,P),V(J,P) * KILL(J,P)) ;
533
534
535  KILLNL(J,P)..
536
537      KILL(J,P) =E= ( (T(J)/C(J)) - D(J,P) ) * ( 1 - EXPO(J,P) ) ;
538
539
540  EXPONENT(J,P)..
541
542      EXPO(J,P) =E= EXP( ((-C(J))/T(J)) * SUM((I,R,W)$B(I,J,R,W),
543
544      E(I,J,R) * SX(I,J,R,W,P)$B(I,J,R,W)) ) ;
545
546
547  DEADTG(J,P)..
548
549      D(J,P) =E= SUM(PP$(ORD(PP) LT ORD(P)),KILL(J,PP) - REBUILD(J,PP)) ;
550
551
552  KILLCON(J,P)..
553
554      KILL(J,P) =L= T(J) - D(J,P) ;
555
556

```

```

557 RECCON(J,P)..
558
559         SUM(PP$(ORD(PP) LE ORD(P)),REBUILD(J,PP)) =L=
560
561         SUM(PP$(ORD(PP) LE PERUP(J,P)),KILL(J,PP) ) ;
562
563
564 SORTCON(I,P)..
565
566         SUM((J,R,W)$B(I,J,R,W),
567
568         SX(I,J,R,W,P)$B(I,J,R,W)) =L= PROP(I,P) * S(I,P) ;
569
570
571 WEAPCONSUM(K)..
572
573         WEAP(K) =E= SUM((I,J,R,W,P)$((ORD(K) EQ WPTYPE(I,J,R)) AND
574
575         (B(I,J,R,W) NE 0) ),B(I,J,R,W) * SX(I,J,R,W,P)$B(I,J,R,W)) ;
576
577
578         SELECT(I,J,R,W)$B(I,J,R,W)..
579
580         0 =E= SUM(P,SX(I,J,R,W,P)$B(I,J,R,W)) *
581
582         SUM((K,RP)$((ORD(RP) LT ORD(R)) AND
583
584         (B(I,J,RP,W) NE 0) AND (ORD(K) EQ WPTYPE(I,J,RP)) ),
585
586         (WP(K) - WEAP(K)) ) ;
587
588
589 DISTR(I,J,W,P)$SUM(R,B(I,J,R,W))..
590
591         SUM(R,SX(I,J,R,W,P)$B(I,J,R,W)) =E= PR(W) *
592
593         SUM((R,WPP)$B(I,J,R,WPP),SX(I,J,R,WPP,P)$B(I,J,R,WPP)) ;
594
595
596
597 MODEL AIRATTACK /ALL/ ;
598
599
600 * Limit for number of iterations
601
602 OPTION ITERLIM = 1000 , LIMCOL = 0 , LIMROW = 0 ;
603
604 OPTION SOLPRINT = OFF , SYSOUT = OFF ;
605
606
607 SOLVE AIRATTACK USING NLP MAXIMIZING Z ;
608
609
610 ** The following statements represent the solution values ;
611
612 PARAMETERS

```

```

613
614 KILLTG(J,P)      number of targets J killed in period P
615 OBJECTIVE(P)     Objective Function Value
616 KILLPOT(J,P)     potential Kill-Capability (target-type vs period)
617 OPSORTIE(I,J,R,P,W) number of optimal sorties
618 SORTIE(J,P,I)    number of sorties I assigned to target J in period P
619 WPCOMB(I,J,K)    number of weapons (sortie , target  and weapon type)
620 WPCONS(R,K)      number of weapons (tactic vs weapon-type)
621 WEAPON(J,K)      number of weapons (target  vs weapon-type)  ;
622
623
624 KILLTG(J,P)  = KILL. L(J,P) ;
625
626 OBJECTIVE(P) = SUM(J,V(J,P) * KILL. L(J,P)) ;
627
628 KILLPOT(J,P) = SUM((I,R,W)$B(I,J,R,W),
629
630 E(I,J,R) * SX. L(I,J,R,W,P)) ;
631
632 WEAPON(J,K) = SUM((I,R,W,P)$ (ORD(K) EQ WPTYPE(I,J,R)),
633
634 B(I,J,R,W) * SX. L(I,J,R,W,P)) ;
635
636 WPCONS(R,K) = SUM((I,J,W,P)$ (
637
638 (ORD(K) EQ WPTYPE(I,J,R)) AND (B(I,J,R,W) NE 0) ),
639
640 B(I,J,R,W) * SX. L(I,J,R,W,P)) ;
641
642 OPSORTIE(I,J,R,P,W) = SX. L(I,J,R,W,P) ;
643
644 SORTIE(J,P,I)      = SUM((R,W),SX. L(I,J,R,W,P)) ;
645
646
647
648 OPTION OBJECTIVE: 2      ; DISPLAY OBJECTIVE      ;
649 OPTION KILLTG: 1: 1: 1   ; DISPLAY KILLTG       ;
650 OPTION KILLPOT: 1: 1: 1  ; DISPLAY KILLPOT      ;
651 OPTION OPSORTIE: 1: 2: 1 ; DISPLAY OPSORTIE     ;
652 OPTION SORTIE: 1: 1: 2   ; DISPLAY SORTIE       ;
653 OPTION WPCONS: 1: 1: 1   ; DISPLAY WPCONS       ;
654 OPTION WEAPON: 1: 1: 1   ; DISPLAY WEAPON       ;
655 OPTION WEAP: 1           ; DISPLAY WEAP. L      ;
656 OPTION REBUILD: 1: 1: 1  ; DISPLAY REBUILD. L   ;

```

COMPILATION TIME = 2.140 SECONDS

MODEL STATISTICS SOLVE AIRATTACK USING NLP FROM LINE 607

MODEL STATISTICS

BLOCKS OF EQUATIONS	10	SINGLE EQUATIONS	932
BLOCKS OF VARIABLES	7	SINGLE VARIABLES	1289
NON ZERO ELEMENTS	9758	NON LINEAR N-Z	1889
DERIVATIVE POOL	31	CONSTANT POOL	61
CODE LENGTH	15943		

GENERATION TIME = 65.580 SECONDS

EXECUTION TIME = 67.680 SECONDS

SOLUTION REPORT SOLVE AIRATTACK USING NLP FROM LINE 607

# S O L V E S U M M A R Y

MODEL	AIRATTACK	OBJECTIVE	Z
TYPE	NLP	DIRECTION	MAXIMIZE
SOLVER	MINOS5	FROM LINE	607

\*\*\*\*\* SOLVER STATUS 1 NORMAL COMPLETION  
 \*\*\*\*\* MODEL STATUS 2 LOCALLY OPTIMAL  
 \*\*\*\*\* OBJECTIVE VALUE 1358.0172

RESOURCE USAGE, LIMIT	64.179	1000.000
ITERATION COUNT, LIMIT	639	1000
EVALUATION ERRORS	0	0

M I N O S --- VERSION 5.0 APR 1984  
 = = = =

COURTESY OF B. A. MURTAGH AND M. A. SAUNDERS,  
 DEPARTMENT OF OPERATIONS RESEARCH,  
 STANFORD UNIVERSITY,  
 STANFORD CALIFORNIA 94305 U. S. A.

WORK SPACE NEEDED (ESTIMATE)	--	104191 WORDS.
WORK SPACE AVAILABLE	--	134740 WORDS.
(MAXIMUM OBTAINABLE	--	288878 WORDS.)

EXIT -- OPTIMAL SOLUTION FOUND

MAJOR ITERATIONS	22	
NORM RG / NORM PI	5.752E-08	
TOTAL USED	65.17 UNITS	
MINOS5 TIME	56.27 (INTERPRETER -	9.78)

\*\*\*\*\* REPORT SUMMARY :  
 0 NONOPT  
 0 INFEASIBLE  
 0 UNBOUNDED  
 0 ERRORS

---- 648 PARAMETER OBJECTIVE OBJECTIVE FUNCTION VALUE

PER1 251.30, PER2 333.84, PER3 674.04, PER4 98.84

---- 649 PARAMETER KILLTG NUMBER OF TARGETS J KILLED IN PERIOD P

	PER1	PER2	PER3	PER4
TG5	0.5	1.1	18.0	2.0
TG8	13.0			
TG10				26.3
TG11	9.6			
TG13	2.2		1.7	0.1
TG14	2.0			
TG22				6.0
TG24			1.5	1.3
TG25		26.6	1.1	
TG27			21.0	
TG34			18.0	

---- 650 PARAMETER KILLPOT POTENTIAL KILL-CAPABILITY (TARGET-TYPE VS PERIOD)

	PER1	PER2	PER3	PER4
TG5	0.5	1.1	20.1	2.5
TG8	13.7			
TG10				29.0
TG11	9.8			
TG13	2.4		2.2	0.2
TG14	2.1			
TG22				6.7
TG24			1.9	3.3
TG25		28.9	1.3	
TG27			36.1	
TG34			20.1	

---- 651 PARAMETER OPSORTIE NUMBER OF OPTIMAL SORTIES

INDEX 1 = AC1 INDEX 2 = TG8

	WB2	WB3	WB4	WB5	WB6
OD3.PER1	0.2	1.2	0.6	0.6	5.8

INDEX 1 = AC1 INDEX 2 = TG10

	WB2	WB3	WB4	WB5	WB6
OD1.PER4	4.2	29.4	14.7	14.7	99.9
OD3.PER4					47.1

INDEX 1 = AC1 INDEX 2 = TG11

	WB2	WB3	WB4	WB5	WB6
OD1. PER1	2.0	14.0	7.0	7.0	69.8

INDEX 1 = AC1 INDEX 2 = TG13

	WB2	WB3	WB4	WB5	WB6
OD1. PER3	0.2	1.4	0.7	0.7	7.2

INDEX 1 = AC1 INDEX 2 = TG25

	WB2	WB3	WB4	WB5	WB6
OD1. PER2	2.0	14.0	7.0	7.0	70.0
OD1. PER3	0.2	1.3	0.7	0.7	6.5

INDEX 1 = AC1 INDEX 2 = TG27

	WB2	WB3	WB4	WB5	WB6
OD1. PER3		5.4	2.7	2.7	27.2
OD3. PER3	0.8				

INDEX 1 = AC1 INDEX 2 = TG34

	WB2	WB3	WB4	WB5	WB6
OD1. PER3	0.9	6.5	3.3	3.3	32.5

INDEX 1 = AC2 INDEX 2 = TG5

	WB2	WB3	WB4	WB5	WB6
OD1. PER1	4.3313E-2	0.3	0.2	0.2	1.5
OD1. PER2	8.6886E-2	0.6	0.3	0.3	3.0
OD1. PER3	1.6	11.4	5.7	5.7	56.9
OD1. PER4	0.2	1.4	0.7	0.7	7.0

INDEX 1 = AC2 INDEX 2 = TG8

	WB2	WB3	WB4	WB5	WB6
OD1. PER1	0.9	6.0	3.0	3.0	29.9

INDEX 1 = AC2 INDEX 2 = TG10

	WB2	WB3	WB4	WB5	WB6
OD3. PER4	3.1	21.9	10.9	10.9	109.4

INDEX 1 = AC2 INDEX 2 = TG11

	WB2	WB3	WB4	WB5	WB6
--	-----	-----	-----	-----	-----



OD1. PER1	0.4	2.9	1.5	1.5	14.6
-----------	-----	-----	-----	-----	------

INDEX 1 = AC2 INDEX 2 = TG13

	WB2	WB3	WB4	WB5	WB6
OD1. PER1	0.2	1.7	0.8	0.8	8.4
OD1. PER4	1.7758E-2	0.1	6.2153E-2	6.2153E-2	0.6

INDEX 1 = AC2 INDEX 2 = TG14

	WB2	WB3	WB4	WB5	WB6
OD1. PER1		0.4	0.2	0.2	2.2
OD2. PER1	6.1764E-2				

INDEX 1 = AC2 INDEX 2 = TG22

	WB2	WB3	WB4	WB5	WB6
OD1. PER4	0.5	3.7	1.9	1.9	18.7

INDEX 1 = AC2 INDEX 2 = TG24

	WB2	WB3	WB4	WB5	WB6
OD1. PER3	0.1	1.0			
OD1. PER4	0.3	2.3	1.1	1.1	11.3
OD3. PER3			0.5	0.5	5.1

INDEX 1 = AC2 INDEX 2 = TG25

	WB2	WB3	WB4	WB5	WB6
OD1. PER2	2.3	16.2	8.1	8.1	81.0

INDEX 1 = AC2 INDEX 2 = TG34

	WB2	WB3	WB4	WB5	WB6
OD1. PER3		2.3	1.1	1.1	11.4
OD3. PER3	0.3				

---- 652 PARAMETER SORTIE

NUMBER OF SORTIES I ASSIGNED TO TARGET J  
IN PERIOD P

	PER1. AC1	PER1. AC2	PER2. AC1	PER2. AC2	PER3. AC1	PER3. AC2
TG5		2.2		4.3		81.3
TG8	8.3	42.8				
TG11	99.7	20.9				
TG13		12.1			10.3	
TG14		3.1				
TG24						7.4
TG25			100.0	115.7	9.3	



TG27		38.8	
TG34		46.5	16.3

+      PER4. AC1      PER4. AC2

TG5		10.0
TG10	210.0	156.3
TG13		0.9
TG22		26.7
TG24		16.1

----- 653    PARAMETER    WPCONS                    NUMBER OF WEAPONS (TACTIC VS WEAPON-TYPE)

	WP1	WP2	WP3	WP5	WP7	WP12
OD1	598.0	586.8	507.3	587.6		76.2
OD2						0.2
OD3	2.0		4.7	12.4	423.5	

+      WP46

OD1      450.0

----- 654    PARAMETER    WEAPON                    NUMBER OF WEAPONS (TARGET VS WEAPON-TYPE)

	WP1	WP2	WP3	WP5	WP7	WP12
TG5		586.8				
TG8	256.6				16.7	
TG10				325.8	406.9	
TG11				241.2		
TG13	77.7			20.7		
TG14						12.4
TG22	160.0					
TG24	103.7			12.4		
TG27			233.0			
TG34	2.0		279.0			64.1

+      WP46

TG25      450.0

----- 655    VARIABLE    WEAP. L

WP1	600.0,	WP2	586.8,	WP3	511.9,	WP5	600.0,	WP7	423.5
WP12	76.4,	WP46	450.0						

----- 656    VARIABLE    REBUILD. L

	PER1	PER2	PER3
TG5	0.5	1.1	2.0

TG11	0.5	1.0
TG25	5.3	10.0

EXECUTION TIME        =        22.580 SECONDS

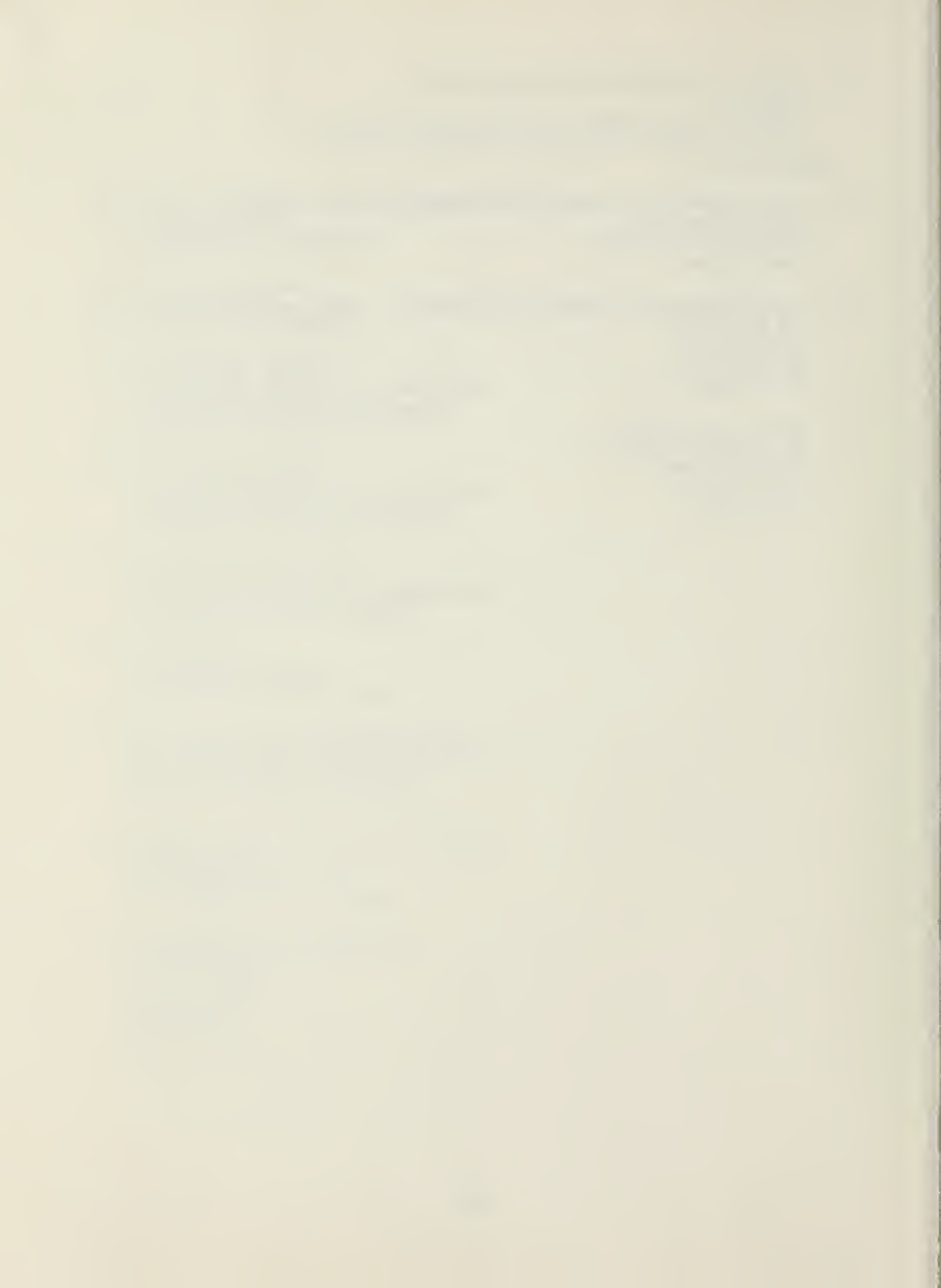
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Thesis

W6519 Wirths

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model for optimized  
sortie allocation.



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